CS195: Computer Vision

Image Transformation

Parametric Warping

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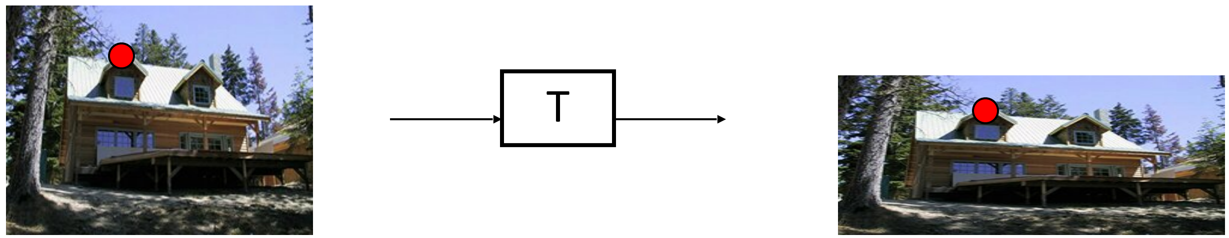
Parametric (global) warping



p = (x,y) p' = (x',y')

Transformation T is a coordinate-changing machine:

p' = *T*(p)

What does it mean that *T* is global?

- Is the same for any point p
- can be described by just a few numbers (parameters)

For linear transformations, we can represent T as a matrix

p' = **M**p

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{M} \begin{bmatrix} x \\ y \end{bmatrix}$$

Parametric (global) warping

Examples of parametric warps:

translation

rotation

aspect

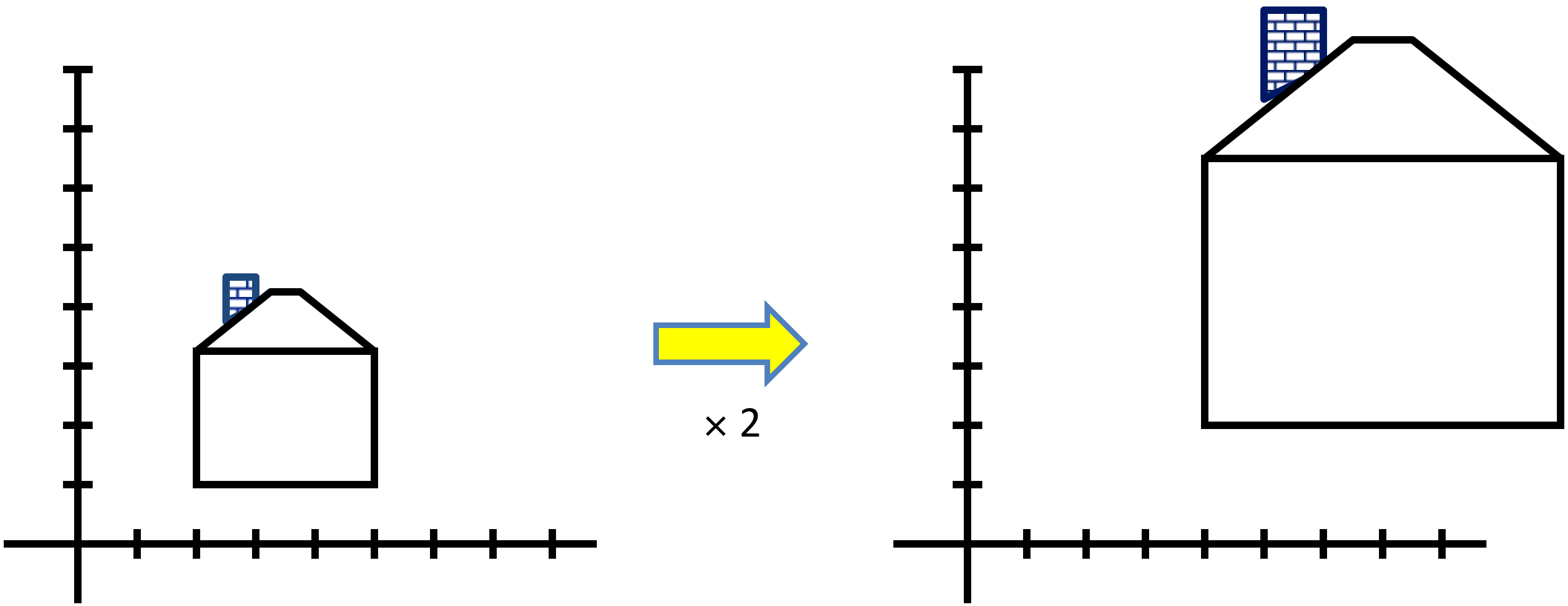
affine

perspective

cylindrical

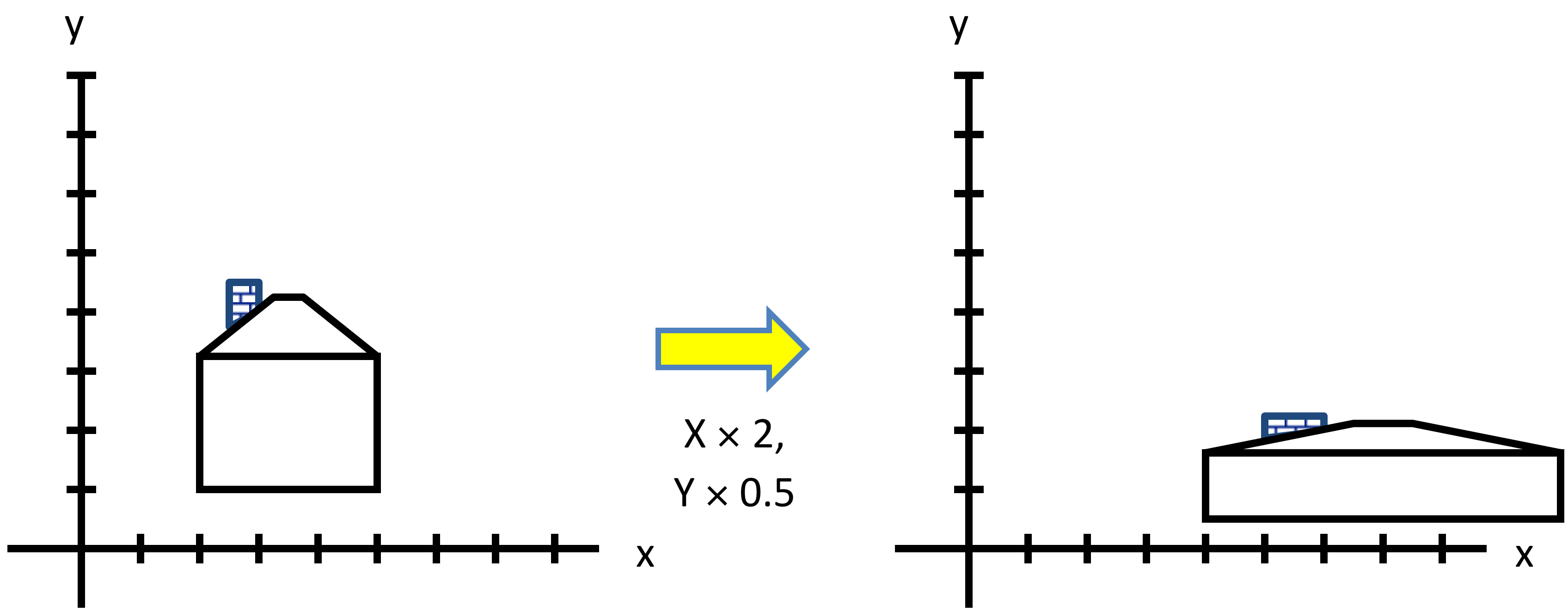
Scaling

- *Scaling* a coordinate means multiplying each of its components by a scalar
- *Uniform scaling* means this scalar is the same for all components:



Scaling

- *Non-uniform scaling* has different scalars per component:



Scaling

- Scaling operation: $$x' = ax$$ $$y' = by$$

- Or, in matrix form:

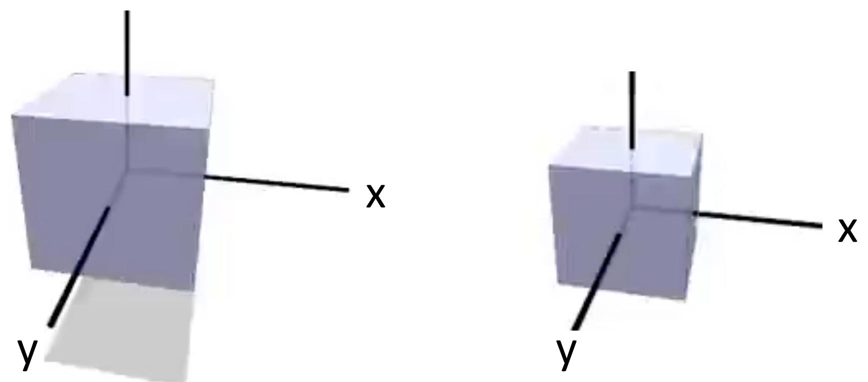


Fig. Viewing from the side

- In order to see the 2D scaling, we need to observe the top surface from a top-down perspective on the x-y plane.

What is the transformation from (x’, y’) to (x, y)?

2x2 Transformation Matrices

- What types of transformations can be represented with a 2x2 matrix?

2D Identity?

$$x' = x$$
$$y' = y$$

$$\begin{array}{ccc} & \text{Matrix} & \text{Vector} \\ \begin{bmatrix} x' \\ y' \end{bmatrix} = & \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & \begin{bmatrix} x \\ y \end{bmatrix} \end{array}$$

2D Scale around (0,0)?

$$x' = s_x * x$$
$$y' = s_y * y$$

$$\begin{array}{ccc} & \text{Matrix} & \text{Vector} \\ \begin{bmatrix} x' \\ y' \end{bmatrix} = & \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} & \begin{bmatrix} x \\ y \end{bmatrix} \end{array}$$

2x2 Transformation Matrices

- What types of transformations can be represented with a 2x2 matrix?

2D Mirror about Y axis?

$$x' = -x$$
$$y' = y$$

$$\begin{array}{ccc} & \textbf{Matrix} & \textbf{Vector} \\ \begin{bmatrix} x' \\ y' \end{bmatrix} = & \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} & \begin{bmatrix} x \\ y \end{bmatrix} \end{array}$$

2D Mirror over (0,0)?

$$x' = -x$$
$$y' = -y$$

$$\begin{array}{ccc} & \textbf{Matrix} & \textbf{Vector} \\ \begin{bmatrix} x' \\ y' \end{bmatrix} = & \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} & \begin{bmatrix} x \\ y \end{bmatrix} \end{array}$$

2x2 Transformation Matrices

- What types of transformations can be represented with a 2x2 matrix?

2D Rotate around (0,0)?

- Below, the figure shows a 3D cube. We are observing the top surface from a top-down perspective on the x-y plane.

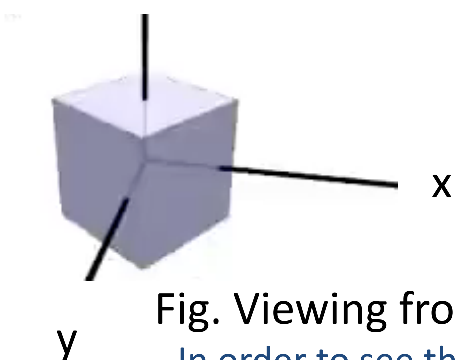
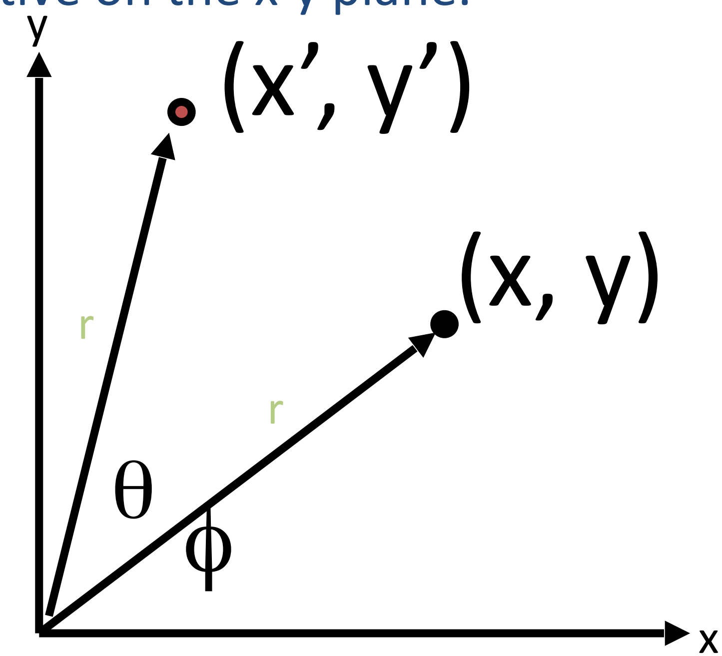


Fig. Viewing from the side

- In order to see the 2D rotation, we need to observe the top surface from a top-down perspective on the x-y plane.



2-D Rotation

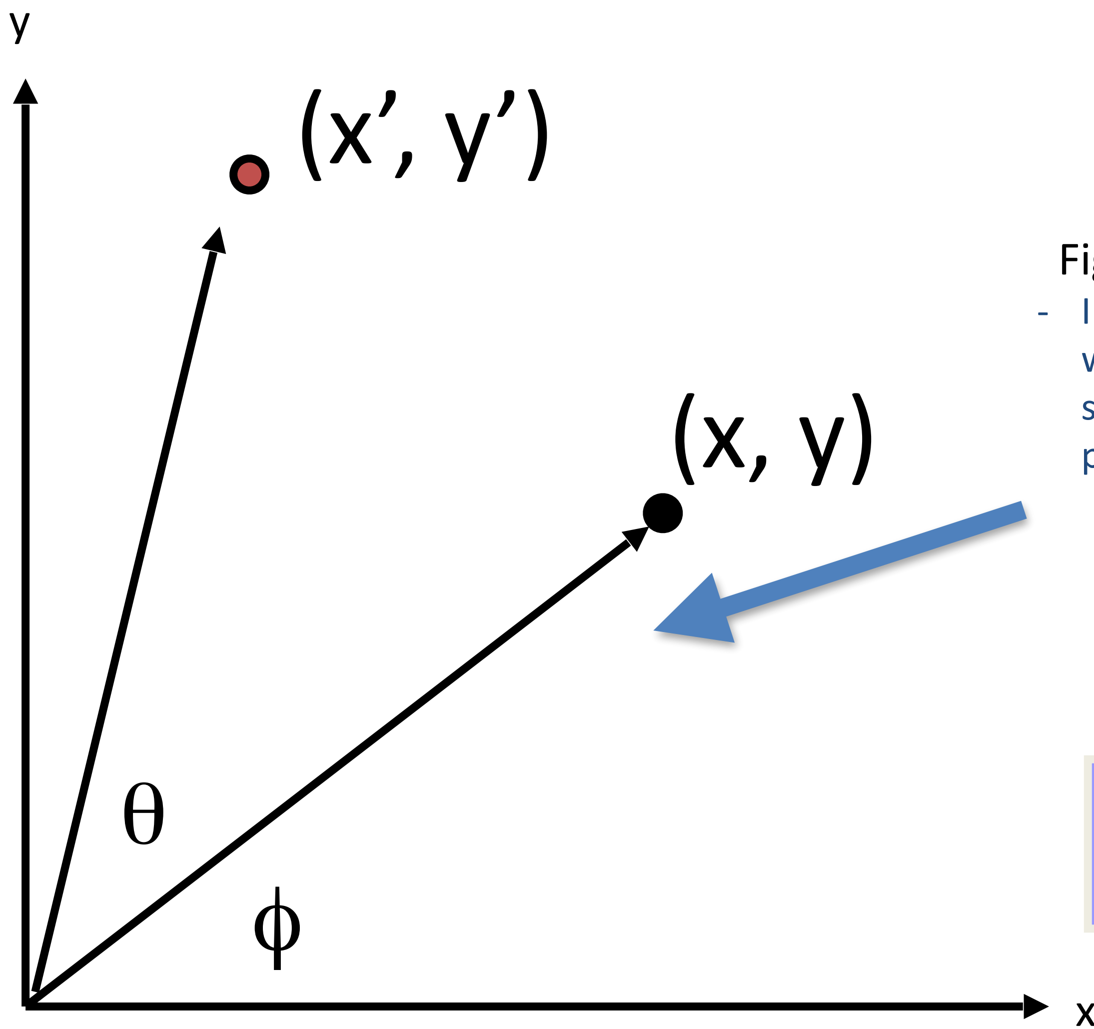


Fig. Viewing from the top

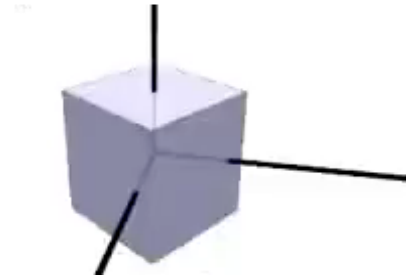
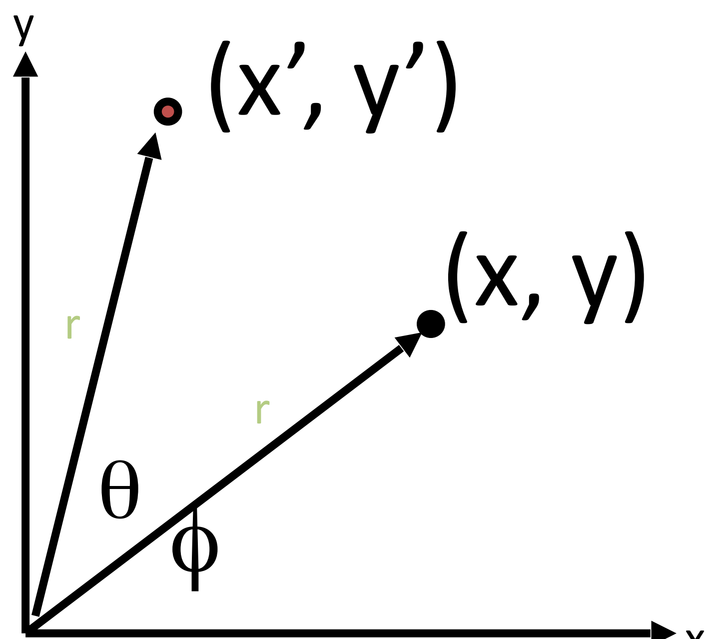


Fig. Viewing from the side

- In order to see the 2D rotation, we need to observe the top surface from a top-down perspective on the x-y plane.

$$x' = x\cos(\theta) - y\sin(\theta)$$
$$y' = x\sin(\theta) + y\cos(\theta)$$

2-D Rotation



Polar coordinates…

$x = r \cos(\phi)$

$y = r \sin(\phi)$

$x' = r \cos(\phi + \theta)$

$y' = r \sin(\phi + \theta)$

Trig Identity…

$$\begin{aligned} x' &= r \cos(\phi + \theta) \\ &= r \cos(\phi) \cos(\theta) - r \sin(\phi) \sin(\theta) \\ &= r \cos(\phi) \cos(\theta) - r \sin(\phi) \sin(\theta) \end{aligned}$$

$$\begin{aligned} y' &= r \sin(\phi + \theta) \\ &= r \sin(\phi) \cos(\theta) + r \cos(\phi) \sin(\theta) \\ &= r \sin(\phi) \cos(\theta) + r \cos(\phi) \sin(\theta) \\ &= r \cos(\phi) \sin(\theta) + r \sin(\phi) \cos(\theta) \end{aligned}$$

Substitute…

$x' = x \cos(\theta) - y \sin(\theta)$

$y' = x \sin(\theta) + y \cos(\theta)$

2-D Rotation

From last slide …

x' = x cos(θ) - y sin(θ)

y' = x sin(θ) + y cos(θ)

- This is easy to capture in matrix form:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\overbrace{\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}}^{\text{Matrix}}}_{\substack{\text{R} \\ \text{rotation matrix}}} \overbrace{\begin{bmatrix} x \\ y \end{bmatrix}}^{\text{Vector}}$$

- Even though sin(θ) and cos(θ) are nonlinear functions of θ,
  - x' is a linear combination of x and y
  - y' is a linear combination of x and y
- Inverse transformation, rotation by –θ
  - For rotation matrices, det(R) = 1 and

$$\mathbf{R}^{-1} = \mathbf{R}^{T}$$

2x2 Transformation Matrices

- What types of transformations can be represented with a 2x2 matrix?

  2D Shear?

$$x' = x + sh_x * y$$

$$y' = sh_y * x + y$$

$$\begin{array}{ccc} & \textbf{Matrix} & \textbf{Vector} \\ \begin{bmatrix} x' \\ y' \end{bmatrix} = & \begin{bmatrix} 1 & sh_x \\ sh_y & 1 \end{bmatrix} & \begin{bmatrix} x \\ y \end{bmatrix} \end{array}$$

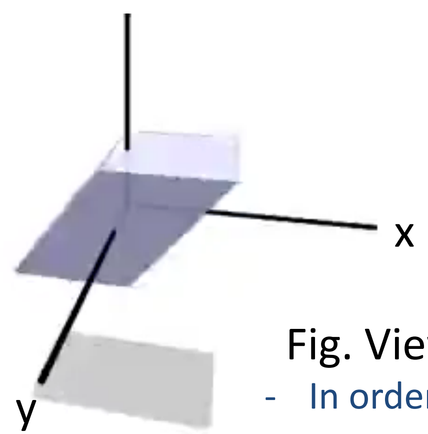


Fig. Viewing from the side

- In order to see the 2D shear, we need to observe the top surface from a top-down perspective on the x-y plane.

2D Linear Transformations

- Any linear transformation can be written as a combination of scale, rotation, shear, and mirror
  - And written as *p'=Mp*

$$\begin{array}{cc} & \textbf{Matrix} \quad \textbf{Vector} \\ \begin{bmatrix} x' \\ y' \end{bmatrix} = & \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \end{array}$$

- Properties of linear transformations:
  - Origin maps to origin
  - Lines map to lines
  - Parallel lines remain parallel
  - Ratios are preserved
  - Closed under composition
  - Cheap to apply

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} i & j \\ k & l \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Linear Transformations

- Any linear transformation can be written as a combination of scale, rotation, shear, and mirror
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$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- Properties of linear transformations:
  - Origin maps to origin
  - Lines map to lines

ALGEBRAIC

x, y

$f(x) + f(x)$ — (i)

$x + y$

$f(x+y)$ — (ii)

GEOMETRIC

X (a) MAPS LINES TO LINE

(b) PRESERVES ORIGIN

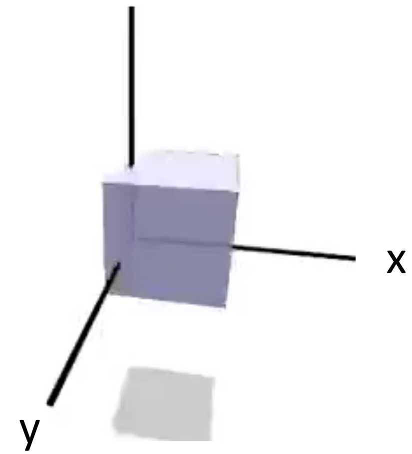
2x2 Matrices

What types of transformations can be represented with a 2x2 matrix?

2D Translation?

$$x' = x + t_x$$

$$y' = y + t_y$$



Only linear 2D transformations can be represented with a 2x2 matrix

2x2 Matrices

What types of transformations can be represented with a 2x2 matrix?

2D Translation?

$$x' = x + t_x$$
$$y' = y + t_y$$

NO!

ADDITIVITY RULE:

$\times$

$f_P(x+y) = x+y+P$

$f_P(x) + f_P(y) = (x+P) + (y+P)$
$= x + y + 2P$

SCALING RULE

$\times$

$f_P(sx) = sx + P$

$s f_P(x) = s(x+P)$
$= sx + sP$

$f(\mathbf{u}+\mathbf{v}) = f(\mathbf{u}) + f(\mathbf{v})$	additivity / operation of addition
$f(c\mathbf{u}) = cf(\mathbf{u})$	homogeneity of degree 1 / operation of scalar multiplication

Group Activity#6

Find the transformed coordinates with given 2x2 transformation matrix

CS195 | In-class activity#6 | Due: Wednesday 11/20/24

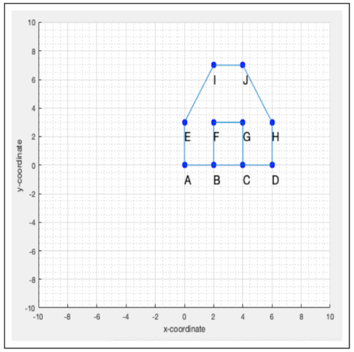
In-class activity#6

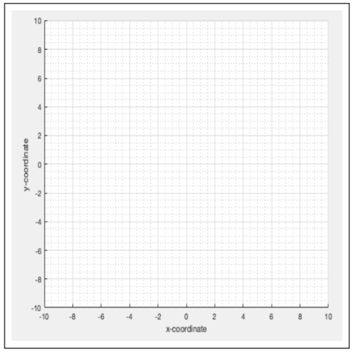
CS 195: Computer Vision, Fall'24

Image Transformation (*6 pts*).

You will be transforming 2D points in this problem. As shown in the Figure below, there are ten 2D coordinates ('A', 'B', ..., 'J') collectively forming a house's shape. The origin is at location *'A'*.

a) Suppose, you are given a 2x2 transformation matrix $\mathbf{M}_1 = \begin{bmatrix} 0.87 & -0.50 \\ 0.50 & 0.87 \end{bmatrix}$. What would be the shape of the house after applying this transformation on the 2D coordinates?





Please write down the transformed coordinates or show them on the empty graph above. What type of transformation does this matrix signify?

Only linear 2D transformations can be represented with a 2x2 matrix