CS181: Computer Vision

2D Image Transformation

Parametric Warping
Linear Transformation
Affine Transformation
Projective Transformation (homography)

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Associate Professor of Computer Science

Parametric (global) warping

Image filtering: change range of image: $g(x) = T(f(x))$

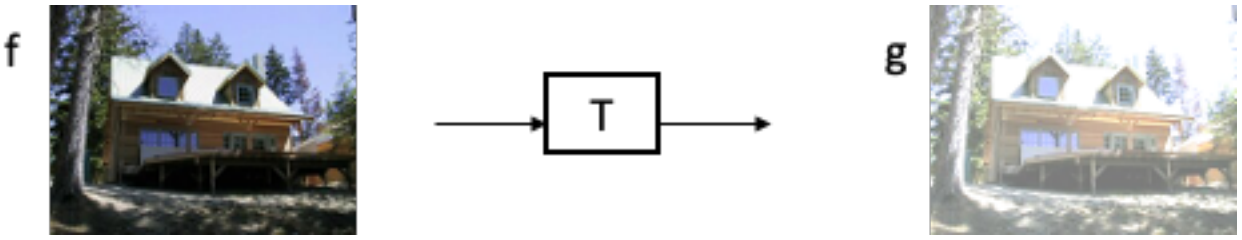
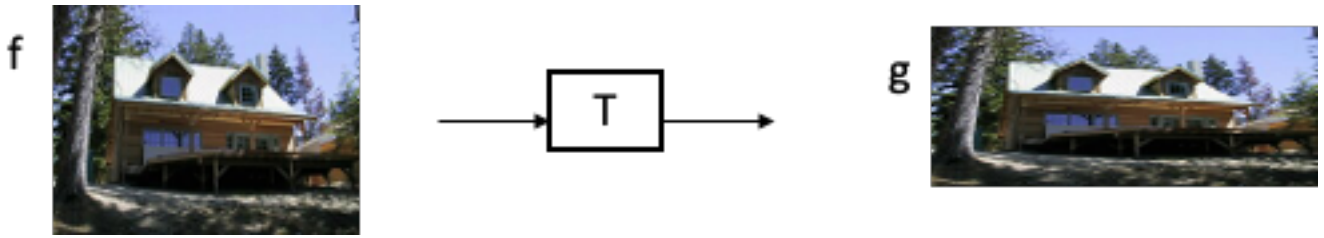
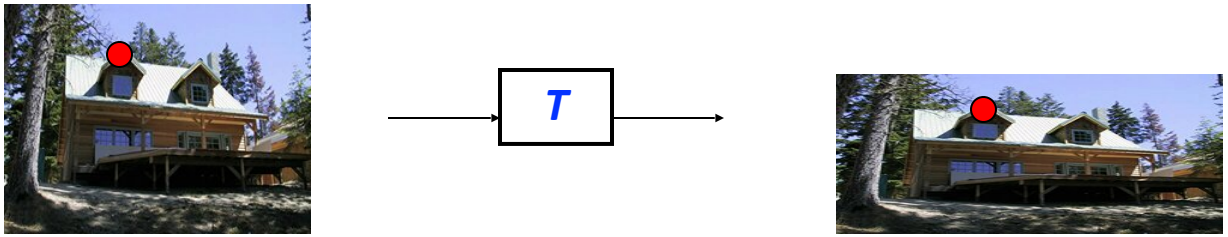


Image warping: change domain of image: $g(x) = f(T(x))$



Parametric (global) warping



p = (x,y)

p’ = (x’,y’)

Transformation **T** is a coordinate-changing machine:

$$p' = T(p)$$

What does it mean that ***T*** is global?

- Is the same for any point p
- can be described by just a few numbers (parameters)

For linear transformations, we can represent ***T*** as a matrix

$$p' = \mathbf{M}p$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{M} \begin{bmatrix} x \\ y \end{bmatrix}$$

Parametric (global) warping

Examples of parametric warps:



spherical warping

Parametric (global) warping

Examples of parametric warps:



translation



rotation



aspect



affine



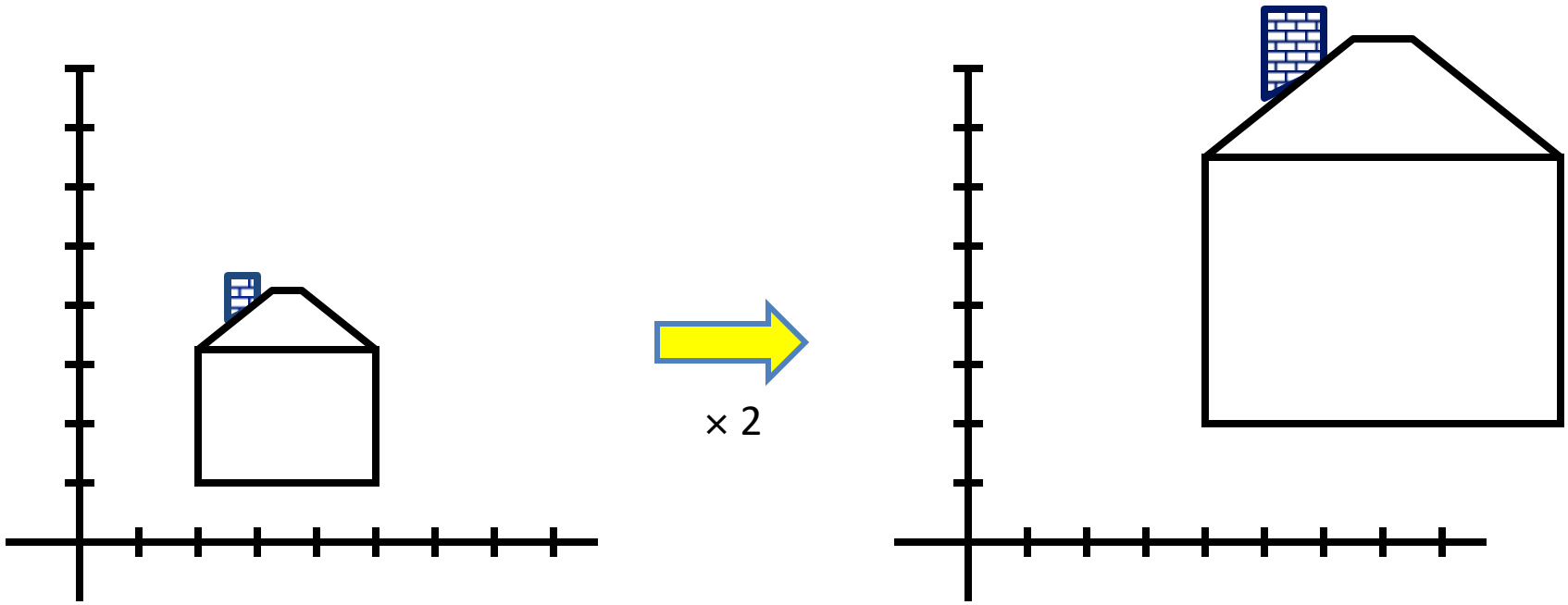
perspective



cylindrical

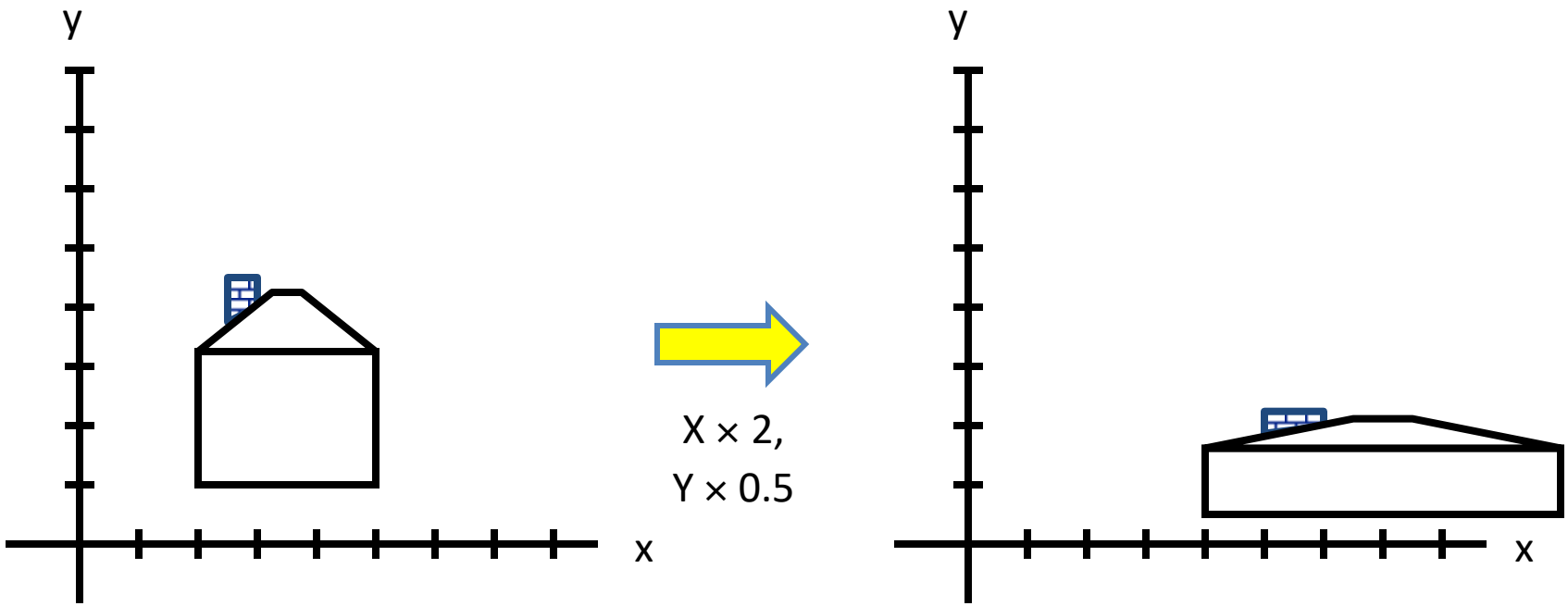
Scaling Transformation

- *Scaling* a coordinate means multiplying each of its components by a scalar
- *Uniform scaling* means this scalar is the same for all components:



Scaling Transformation

- *Non-uniform scaling* has different scalars per component:



Scaling Transformation

- Scaling operation:

$$x' = ax$$

$$y' = by$$

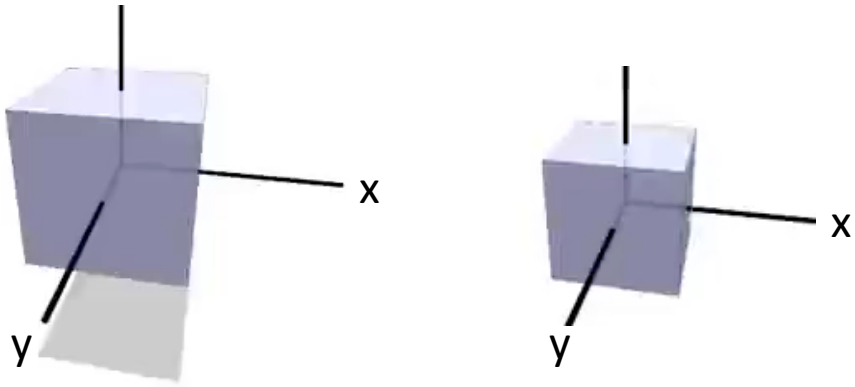


Fig. Viewing from the side

In order to see the 2D scaling, we need to observe the top surface from a top-down perspective on the x-y plane.

What is the transformation from (x', y') to (x, y)?

2x2 Transformation Matrices

What types of transformations can be represented with a **2x2 matrix**?

2D Identity?

$$x' = x$$
$$y' = y$$

Matrix Vector

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2x2 Transformation Matrices

What types of transformations can be represented with a **2x2 matrix**?

2D Scale around (0,0)?

$$x' = s_x * x$$

$$y' = s_y * y$$

Matrix Vector

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2x2 Transformation Matrices

What types of transformations can be represented with a **2x2 matrix**?

2D Mirror about Y axis?

$$x' = -x$$
$$y' = y$$

Matrix Vector

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2x2 Transformation Matrices

What types of transformations can be represented with a **2x2 matrix**?

2D Mirror over (0,0)?

$$x' = -x$$
$$y' = -y$$

Matrix Vector

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2x2 Transformation Matrices

What types of transformations can be represented with a **2x2 matrix**?

2D Shear?

$$x' = x + sh_x * y$$

$$y' = sh_y * x + y$$

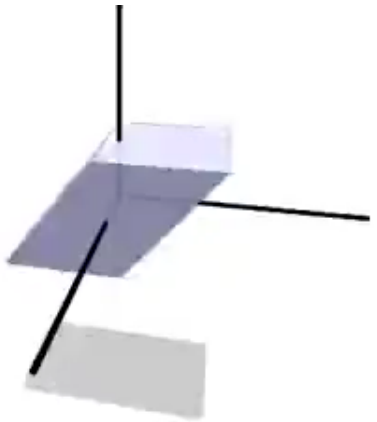


Fig. Viewing from the side

In order to see the 2D shear, we need to observe the top surface from a top-down perspective on the x-y plane.

Matrix Vector

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} 1 & sh_x \\ sh_y & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2x2 Transformation Matrices

What types of transformations can be represented with a **2x2 matrix**?

2D Rotate around (0,0)?

Figure below shows a 3D cube. We need to observe the top surface from a top-down perspective on the x-y plane to visualize the 2D rotation

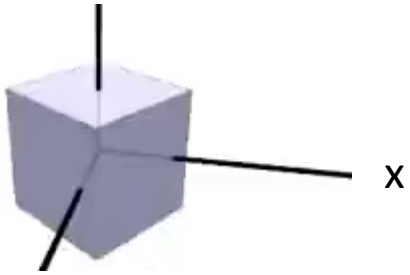


Figure: viewing from the side

In order to see the 2D rotation, we need to observe the top surface from a top-down perspective on the x-y plane.

2x2 Transformation Matrices

What types of transformations can be represented with a **2x2 matrix**?

2D Rotate around (0,0)?

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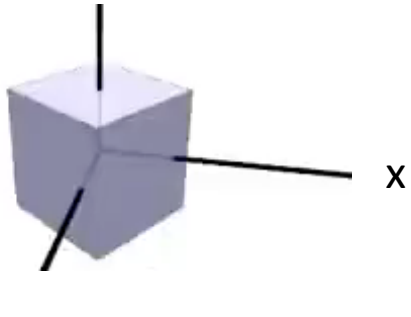
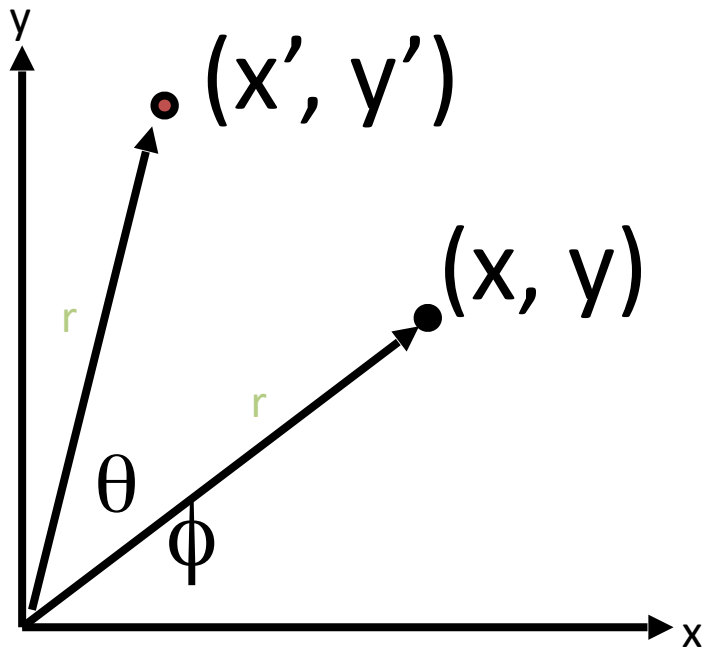


Figure: viewing from the side

In order to see the 2D rotation, we need to observe the top surface from a top-down perspective on the x-y plane.



2x2 Transformation Matrices

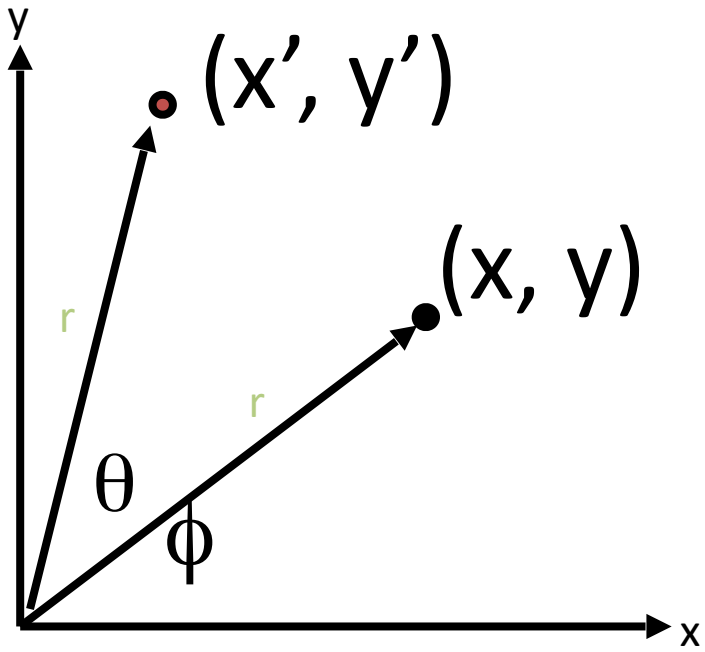
What types of transformations can be represented with a **2x2 matrix**?

2D Rotate around (0,0)?

Figure below shows a 3D cube. We need to observe the top surface from a top-down perspective on the x-y plane to visualize the 2D rotation

Matrix Vector

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



2-D Rotation

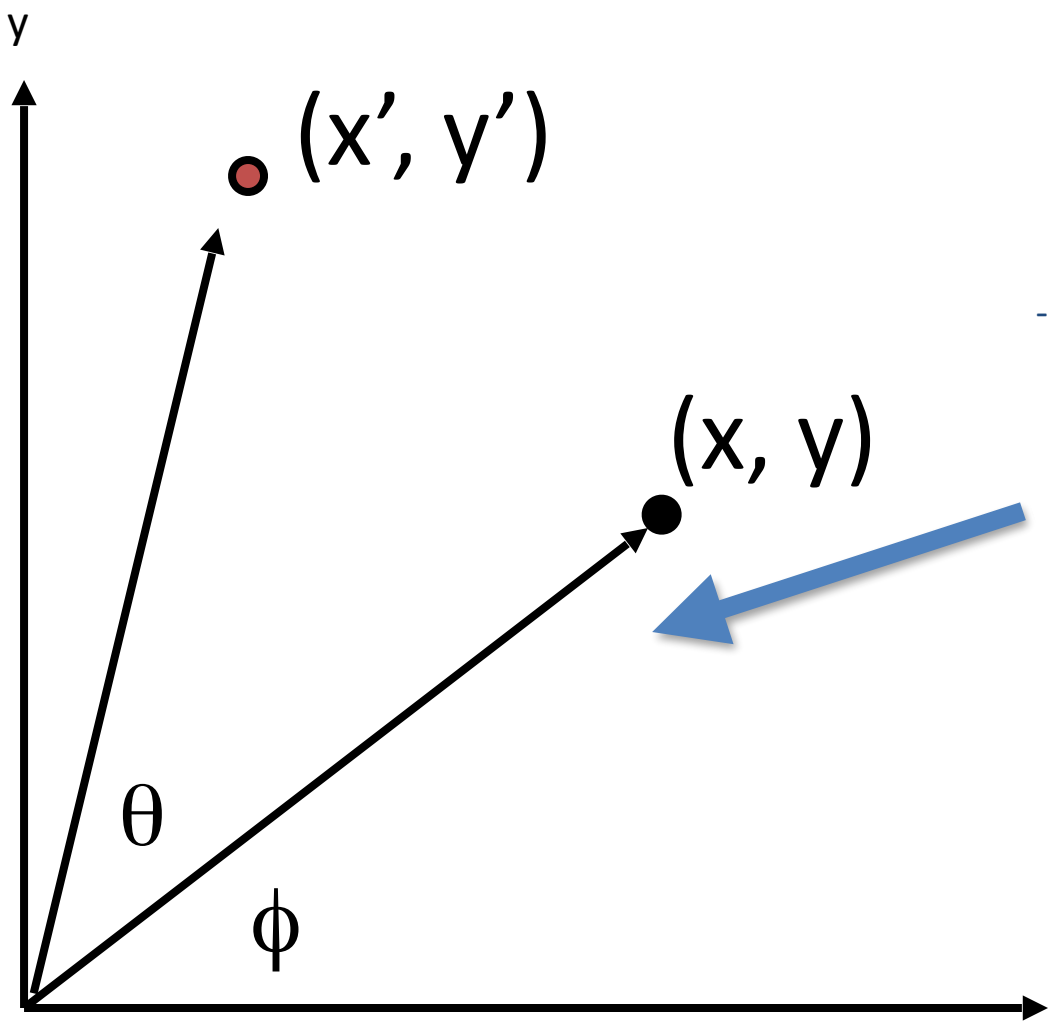


Fig. Viewing from the top

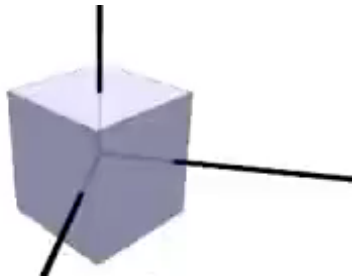
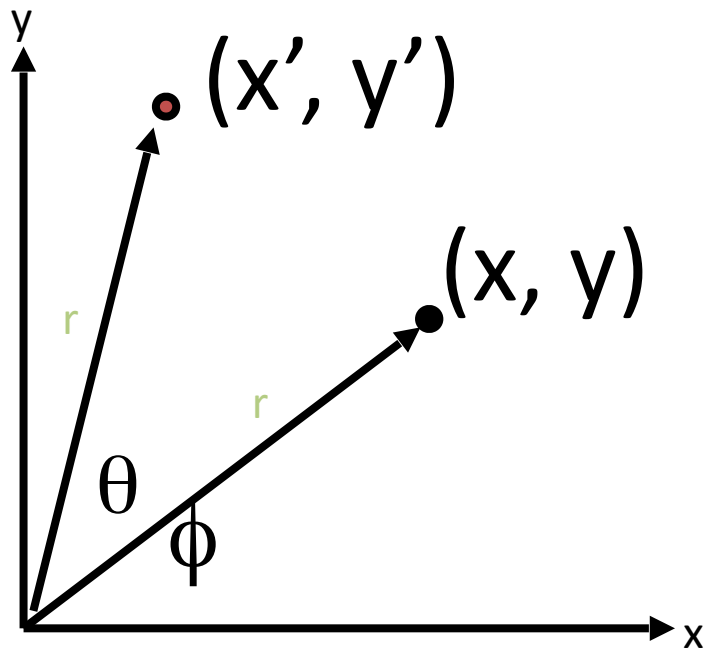


Fig. Viewing from the side

- In order to see the 2D rotation, we need to observe the top surface from a top-down perspective on the x-y plane.

$$x' = x\cos(\theta) - y\sin(\theta)$$
$$y' = x\sin(\theta) + y\cos(\theta)$$

2-D Rotation



Polar coordinates…

$x = r \cos(\phi)$

$y = r \sin(\phi)$

$x' = r \cos(\phi + \theta)$

$y' = r \sin(\phi + \theta)$

Trig Identity…

$x' = r \cos(\phi + \theta)$

$= r \cos(\phi) \cos(\theta) - r \sin(\phi) \sin(\theta)$

$= r \cos(\phi) \cos(\theta) - r \sin(\phi) \sin(\theta)$

$y' = r \sin(\phi + \theta)$

$= r \sin(\phi) \cos(\theta) + r \cos(\phi) \sin(\theta)$

$= r \sin(\phi) \cos(\theta) + r \cos(\phi) \sin(\theta)$

$= r \cos(\phi) \sin(\theta) + r \sin(\phi) \cos(\theta)$

Substitute…

$x' = x \cos(\theta) - y \sin(\theta)$

$y' = x \sin(\theta) + y \cos(\theta)$

2x2 Transformation Matrices

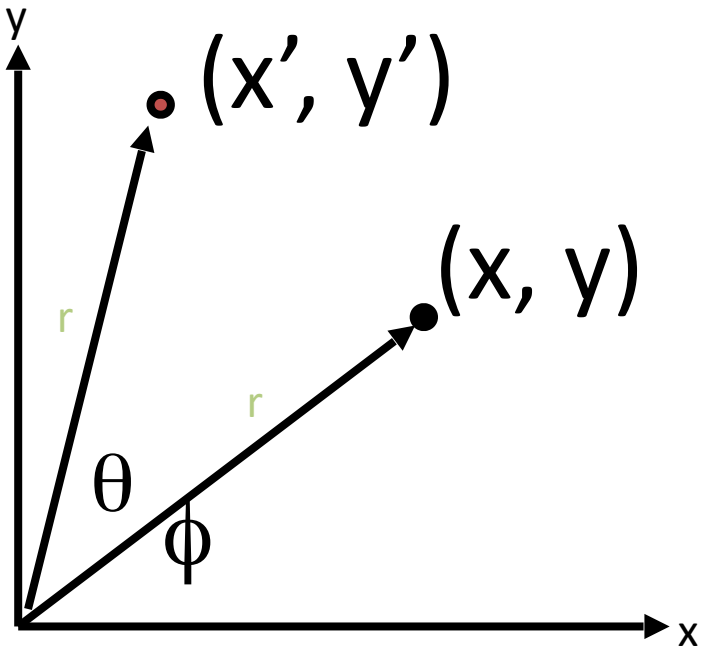
What types of transformations can be represented with a **2x2 matrix**?

2D Rotate around (0,0)?

Figure below shows a 3D cube. We need to observe the top surface from a top-down perspective on the x-y plane to visualize the 2D rotation

Matrix **Vector**

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



2-D Rotation

From last slide …

$x' = x \cos(\theta) - y \sin(\theta)$

$y' = x \sin(\theta) + y \cos(\theta)$

- This is easy to capture in matrix form:

Matrix Vector

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}}_{R} \begin{bmatrix} x \\ y \end{bmatrix}$$

R
rotation matrix

- Even though $\sin(\theta)$ and $\cos(\theta)$ are nonlinear functions of θ,
  - x' is a linear combination of x and y
  - y' is a linear combination of x and y
- Inverse transformation, rotation by $-\theta$
  - For rotation matrices, det(R) = 1 and

$$\mathbf{R}^{-1} = \mathbf{R}^{T}$$

2D Linear Transformations

- Any **linear transformations** can be written as a combinations of (rotation, scaling, shear, mirror)
  - And written as $p' = Mp$

Matrix Vector

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- Properties of linear transformations:
  - Origin maps to origin
  - Lines map to lines
  - Parallel lines remain parallel
  - Ratios are preserved
  - Closed under composition
  - Cheap to apply

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} i & j \\ k & l \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Linear Transformations

- Any linear transformation can be written as a combination of scale, rotation, shear, and mirror
  - And written as *p'=Mp*

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- Properties of linear transformations:
  - Origin maps to origin
  - Lines map to lines

ALGEBRAIC

x, y

$\hookrightarrow f(x) + f(x)$ — (i)

$\hookrightarrow x + y$

$\hookrightarrow f(x+y)$ — (ii)

GEOMETRIC

$\rightarrow$ X (a) MAPS LINES TO LINE

(b) PRESERVES ORIGIN



$f(\mathbf{u}+\mathbf{v}) = f(\mathbf{u}) + f(\mathbf{v})$	additivity / operation of addition
$f(c\mathbf{u}) = cf(\mathbf{u})$	homogeneity of degree 1 / operation of scalar multiplication

2x2 Matrices

Scaling, mirroring, shearing, rotation are all linear transformations

2D Scale around (0,0)?

$$= \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Mirror over (0,0)?

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Shear?

$$= \begin{bmatrix} 1 & sh_x \\ sh_y & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Rotate around (0,0)?

$$= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Only linear 2D transformations
can be represented with a 2x2 matrix

Group Activity#5

Find the transformed coordinates with given 2x2 transformation matrix

CS181 In-class activity#6 Due: Thursday 11/12/26

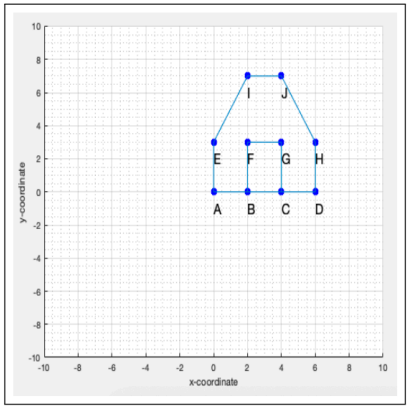
In-class activity#5

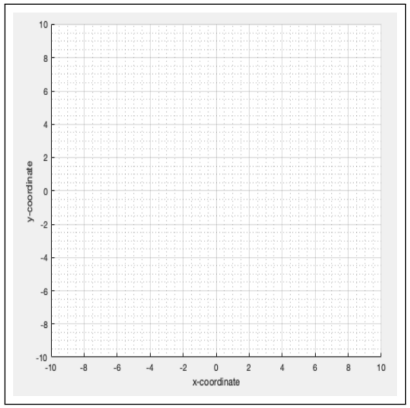
CS 181: Computer Vision, Fall'26

Image Transformation (*6 pts*).

You will be transforming 2D points in this problem. As shown in the Figure below, there are ten 2D coordinates ('A', 'B', ..., 'J') collectively forming a house's shape. The origin is at location *'A'*.

a) Suppose, you are given a 2x2 transformation matrix $\mathbf{M}_1 = \begin{bmatrix} 0.87 & -0.50 \\ 0.50 & 0.87 \end{bmatrix}$. What would be the shape of the house after applying this transformation on the 2D coordinates?

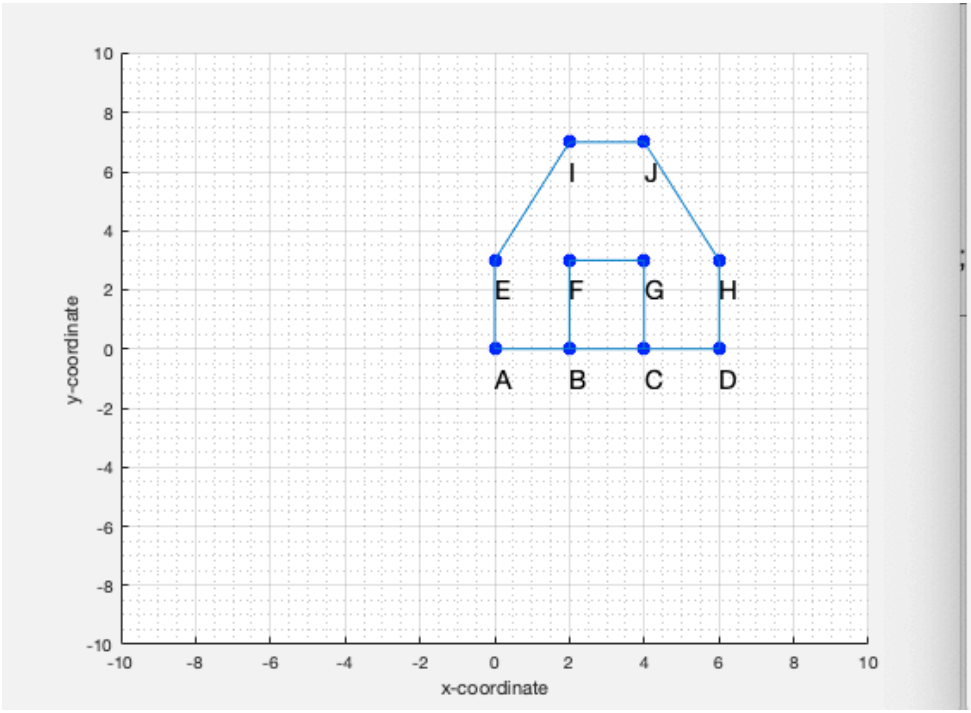


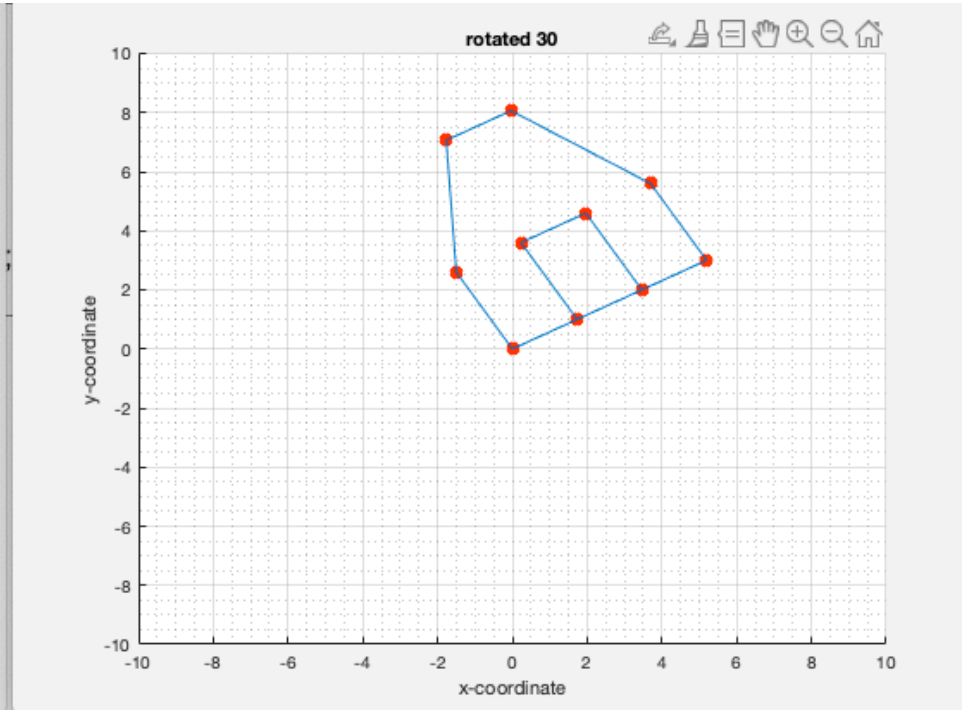


Please write down the transformed coordinates or show them on the empty graph above. What type of transformation does this matrix signify?

Only linear 2D transformations
can be represented with a 2x2 matrix

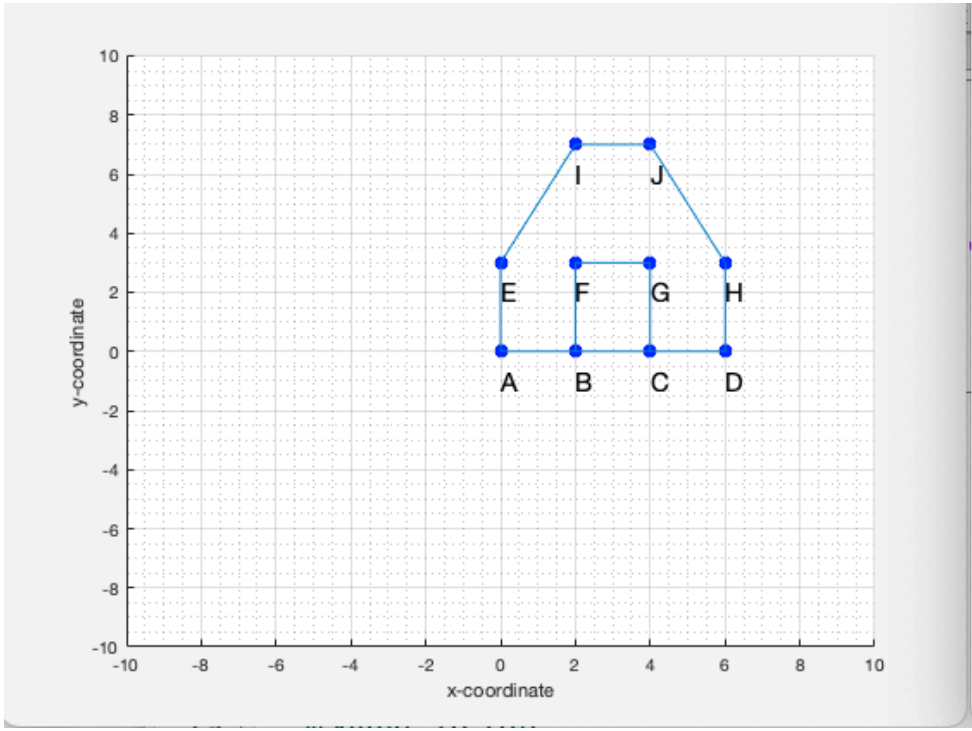
Answers: Group Activity#5

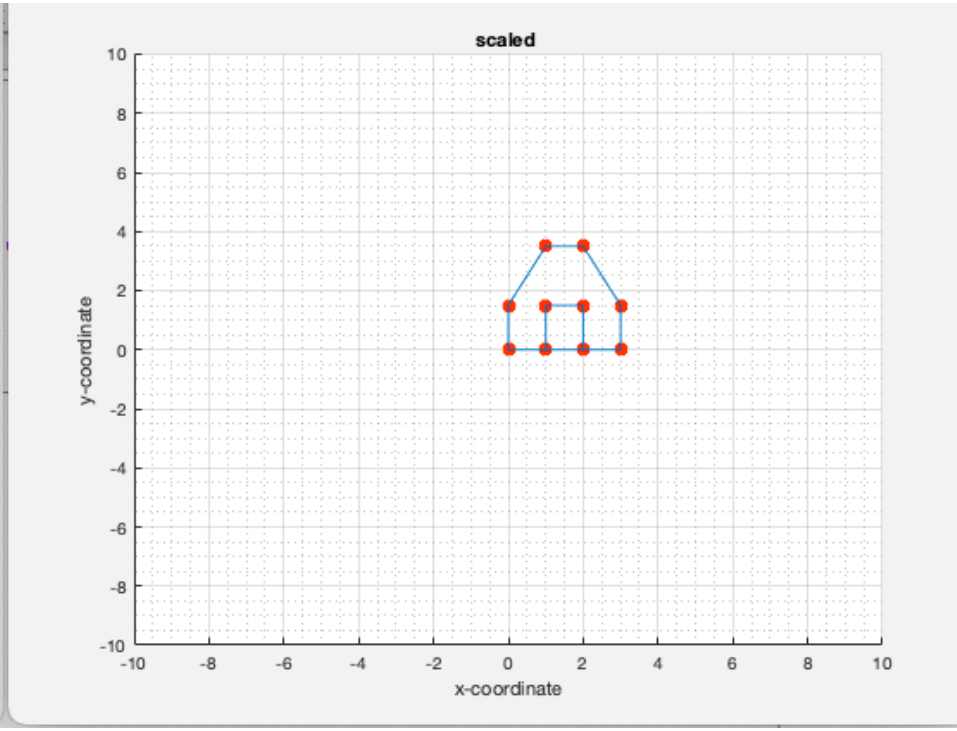




Only linear 2D transformations
can be represented with a 2x2 matrix

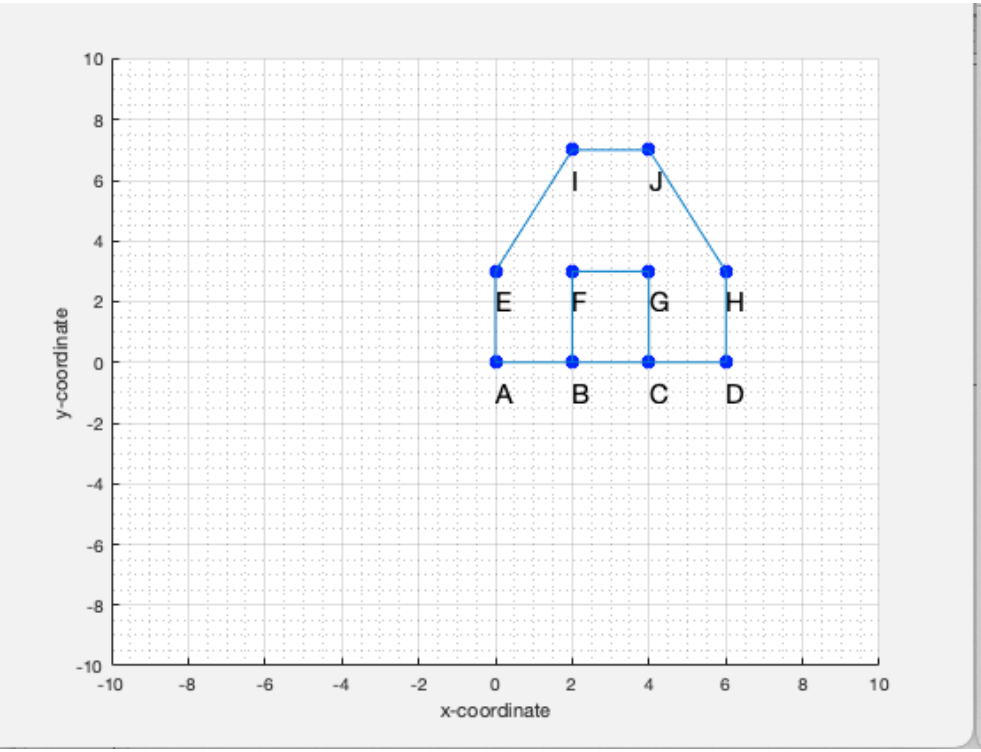
Answers: Group Activity#5

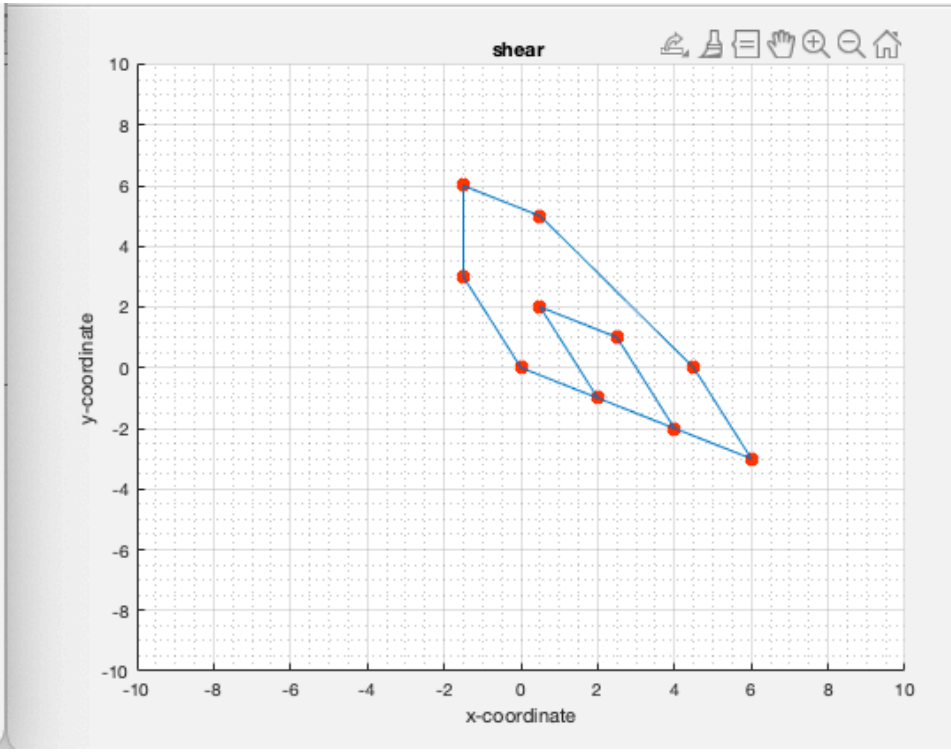




Only linear 2D transformations
can be represented with a 2x2 matrix

Answers: Group Activity#5





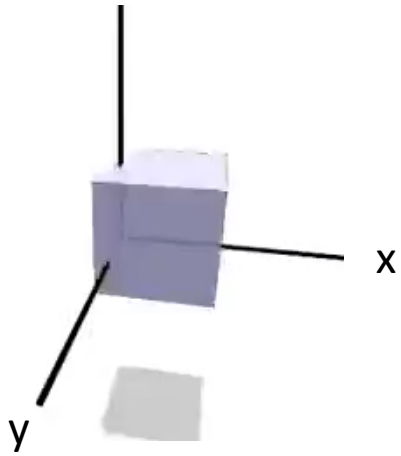
Only linear 2D transformations
can be represented with a 2x2 matrix

2x2 Matrices

Is **2D translation** a linear transformation?

Can we express a 2D translation transformation with a 2x2 matrix?

$$x' = x + t_x$$
$$y' = y + t_y$$



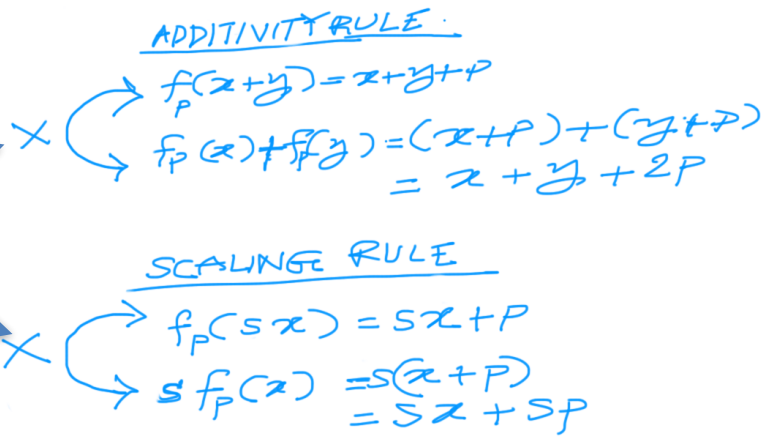
Only linear 2D transformations
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2x2 Matrices

Can we express a 2D translation transformation with a 2x2 matrix?

$$x' = x + t_x$$
$$y' = y + t_y$$

No, we can't
Express it with
a 2x2 matrix!



$f(\mathbf{u}+\mathbf{v}) = f(\mathbf{u}) + f(\mathbf{v})$	additivity / operation of addition
$f(c\mathbf{u}) = cf(\mathbf{u})$	homogeneity of degree 1 / operation of scalar multiplication

Solution: Homogeneous Coordinates

- How can we represent translation as a matrix?

$$x' = x + t_x$$

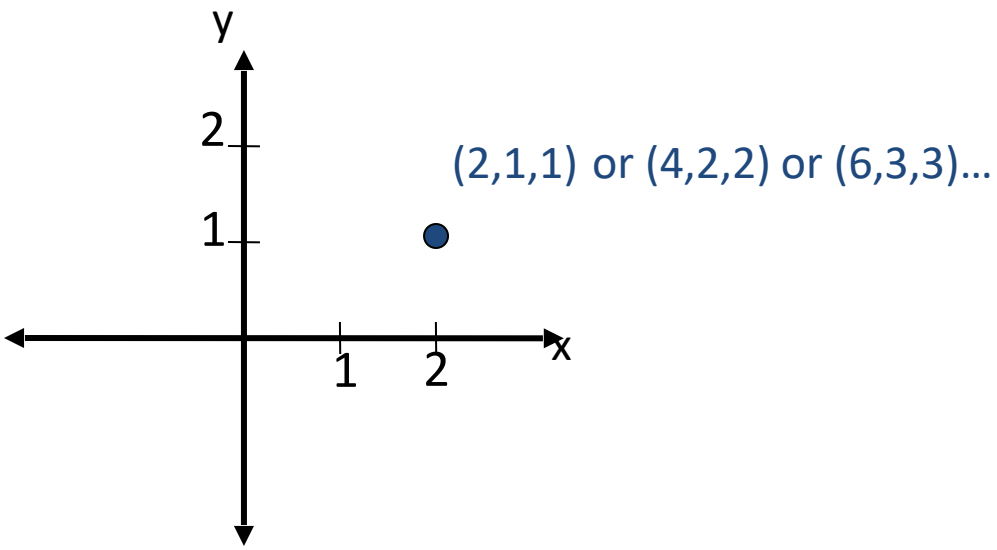
$$y' = y + t_y$$

- Homogeneous coordinates
  - Embed 2-D coordinates in 3-D space

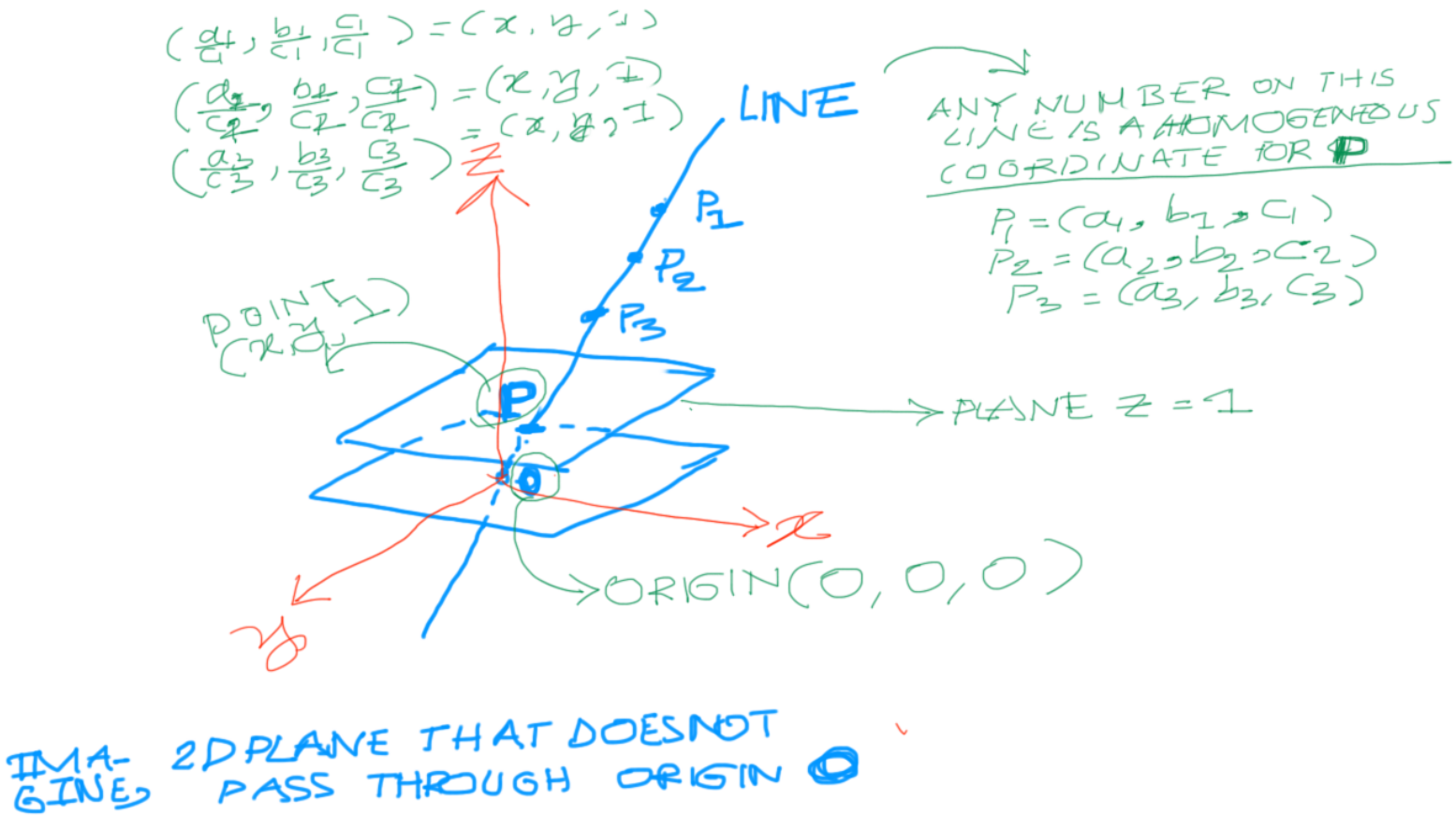
$$\begin{bmatrix} x \\ y \end{bmatrix} \xrightarrow{\text{homogeneous coords}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Homogeneous Coordinates

- Embed in higher-dimensional space. For example:
  - (x, y, w) represents a point at 2D location (x/w, y/w)
  - (x, y, 0) represents a point at infinity
  - (0, 0, 0) is not allowed



Homogeneous Coordinates



Homogeneous Coordinates

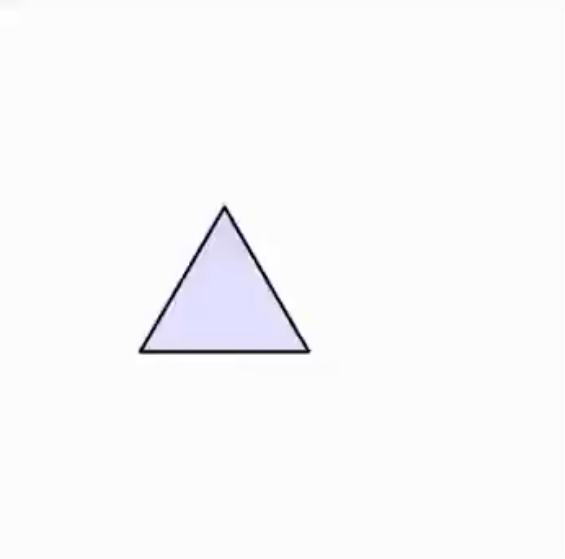


Fig. Bird-eye view or viewing from the top

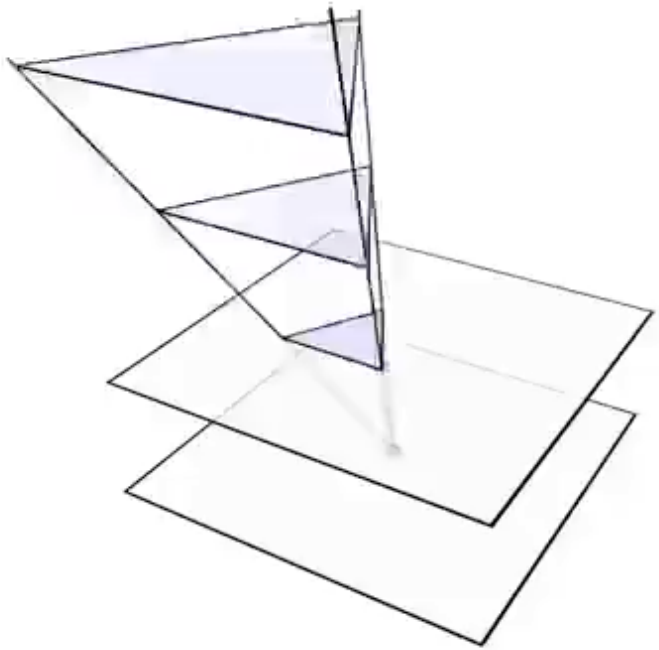


Fig. Viewing from the side

- In order to see the 2D translation, we need to observe the top surface from a top-down perspective on the x-y plane.

Homogeneous Coordinates

- How can we represent translation as matrix?

$$x' = x + t_x$$

$$y' = y + t_y$$

- Use a 3x3 transformation matrix **T**
  - Translation corresponds to last column of matrix

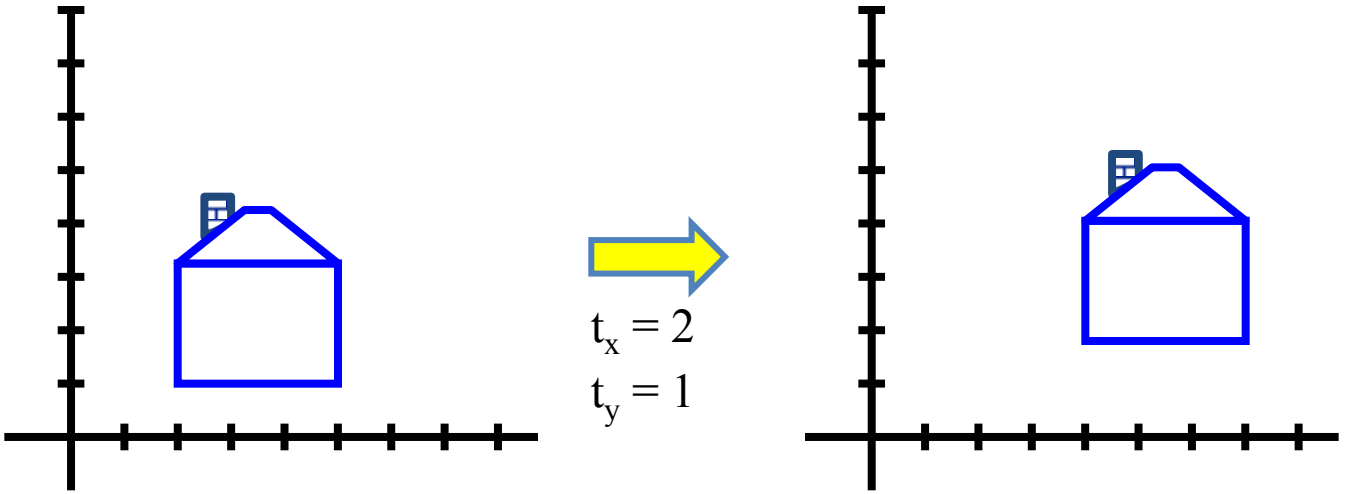
$$\mathbf{T} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

Translation in Homogeneous Coordinates

- Example of translation in homogeneous coordinates

Matrix **Vector (in homogeneous coordinate)**

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$



Homogeneous 2D Transformations

- Basic 2D transformations (others including 2D translation) as 3x3 matrices

Vector (in homogeneous coordinate)

Matrix

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

Numbers can be placed in the **first 6 entries of each** matrix. However, they cannot be placed in the bottom-left two entries.

Vector (in homogeneous coordinate)

Matrix

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Scale

Vector (in homogeneous coordinate)

Matrix

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 \\ \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotate

Vector (in homogeneous coordinate)

Matrix

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & sh_x & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Shear

Affine Transformations

- Affine transformations are combinations of linear transformations (rotation, scaling, shear, mirror) and translation

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

- Properties of affine transformations:
  - Origin does not necessarily map to origin
  - Lines map to lines
  - Parallel lines remain parallel
  - Ratios are preserved
  - Closed under composition
  - Models change of basis
  - Maps triangles to triangles

Affine transformation: when any combination of numbers can be placed in the first six entries of a matrix

Projective Transformations

- Projective transformations are combinations of affine transformations and projective warps

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

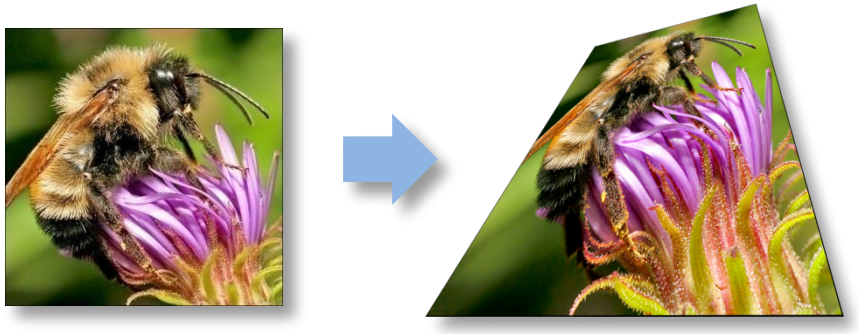
- Properties of projective transformations:
  - Origin does not necessarily map to origin
  - Lines map to lines
  - Parallel lines do not necessarily remain parallel
  - Ratios are not preserved
  - Closed under composition
  - Models change of basis
  - Maps any quadrilateral to any quadrilateral

Projective transformation: when any combination of numbers can be placed in all 9 entries of a matrix

Projective Transformations

$$\mathbf{H} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

Called a *homography* (or on a *flat planar perspective map*)



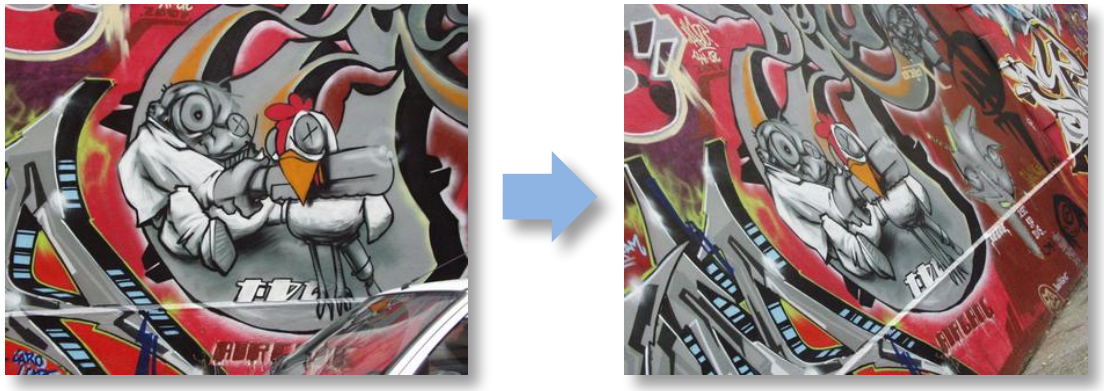
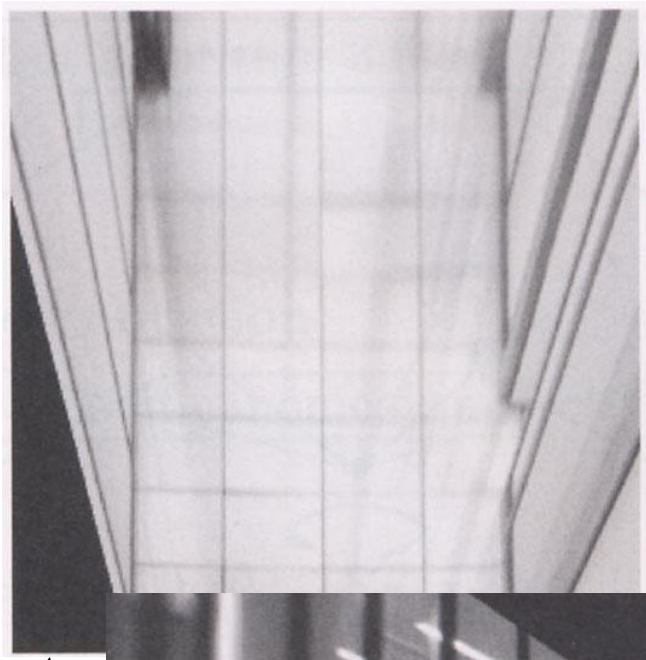
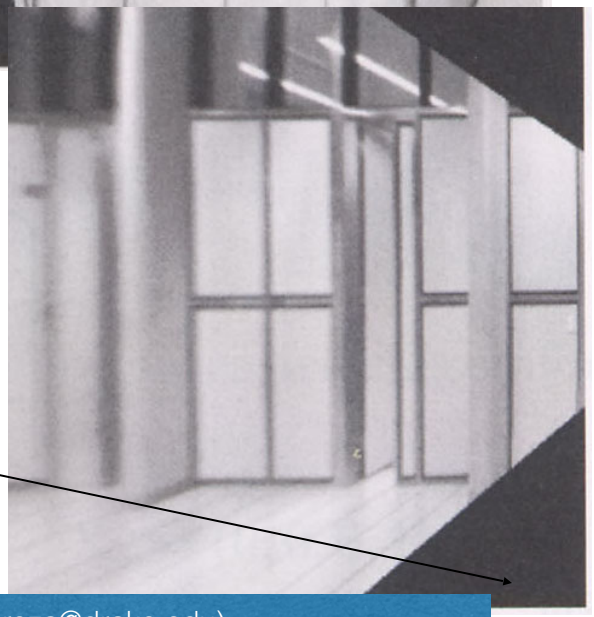


Image warping with homographies







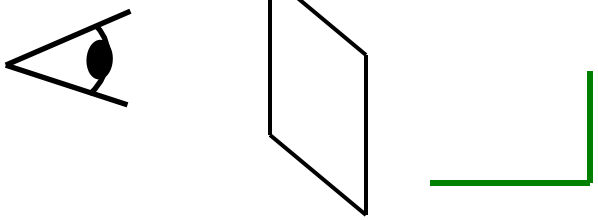
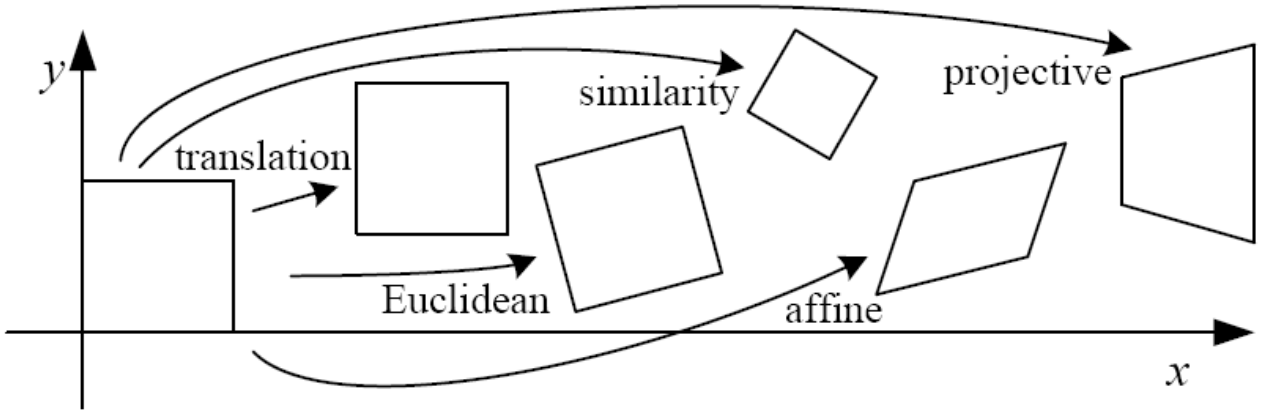


image plane in front

black area
where no pixel
maps to

2D Image Transformations



Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$\begin{bmatrix} \boldsymbol{I} \mid \boldsymbol{t} \end{bmatrix}_{2\times3}$			
rigid (Euclidean)	$\begin{bmatrix} \boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{2\times3}$			
similarity	$\begin{bmatrix} s\boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{2\times3}$			
affine	$\begin{bmatrix} \boldsymbol{A} \end{bmatrix}_{2\times3}$			
projective	$\begin{bmatrix} \tilde{\boldsymbol{H}} \end{bmatrix}_{3\times3}$			

These transformations are a nested set of groups

- Closed under composition and inverse is a member

Matrix Composition

- Transformations can be combined (composed) by matrix multiplication

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \left(\begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 \\ \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} sx & 0 & 0 \\ 0 & sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

$$p' = \mathrm{T}(t_x, t_y) \quad \mathrm{R}(\theta) \quad \mathrm{S}(s_x, s_y) \quad p$$

Matrix Composition

- Rotations can be performed efficiently using a series of three shear operations
  - Horizontal shear with parameter –tan(θ/2)
  - Vertical shear with parameter sin(θ)
  - Horizontal shear with parameter –tan(θ/2) (again)

$$\begin{bmatrix} 1 & -\tan(\theta/2) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \sin(\theta) & 1 \end{bmatrix} \begin{bmatrix} 1 & -\tan(\theta/2) \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

