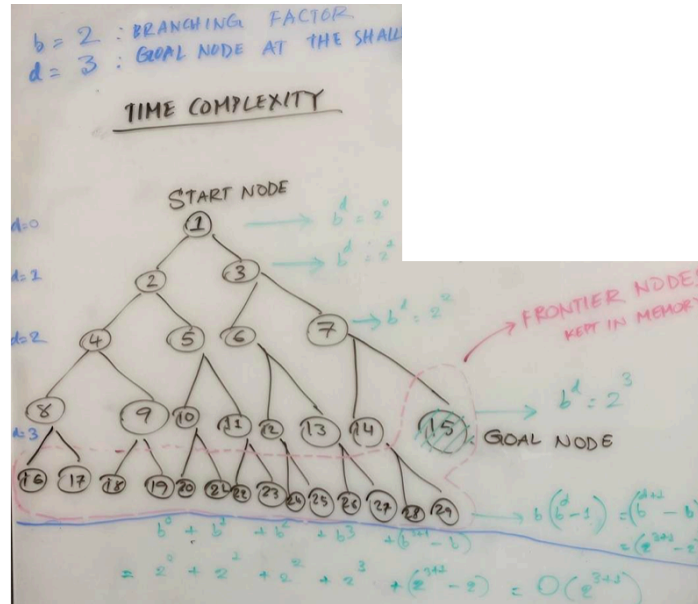


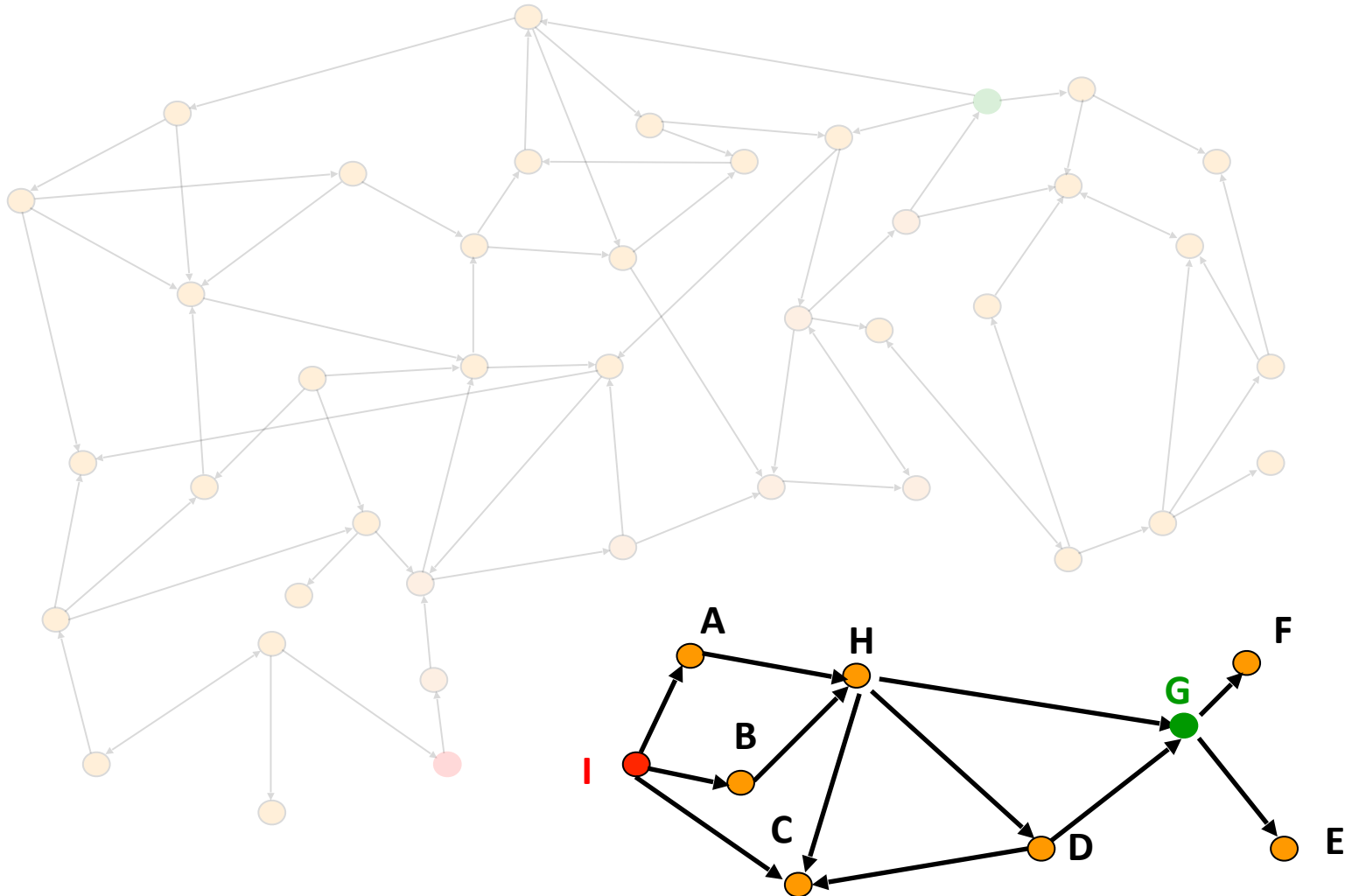
# CS143: Artificial Intelligence



BFS and DFS implementation for **mapbot**  
Analysis of Search Algorithms (BFS, DFS, IDS)



# Recap: Exercise - BFS on a Graph with loop



# Recap: Iterative Deepening Search (IDS)

- Uses a limited version of DFS called **depth-limited search**
  - nodes at depth  $l$  has no successors
- do a **depth-limited search** with  $l = 0$ , if it fails,
- do a **depth-limited search** with  $l = 1$ , if it fails,
- do a **depth-limited search** with  $l = 2$ , if it fails,
  - ...
  - ...
  - ...
- until you find a goal

# Recap: Iterative Deepening Search (IDS)

Limit = 0



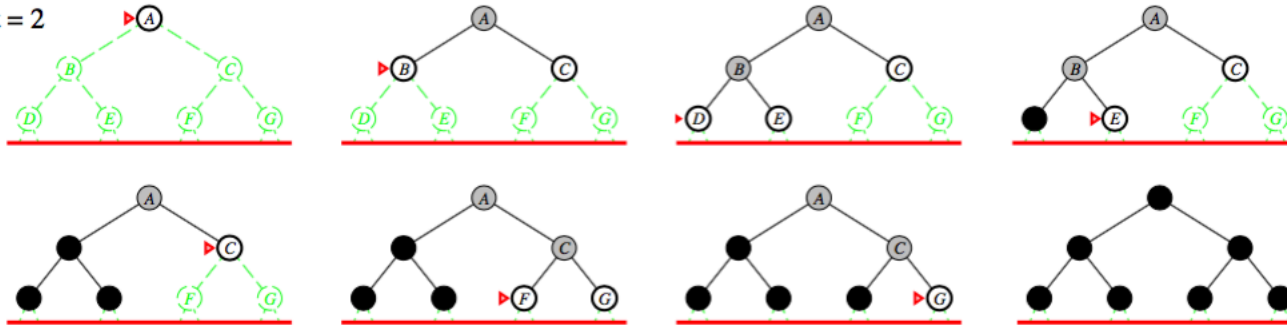
# Recap: Iterative Deepening Search (IDS)

Limit = 1



# Recap: Iterative Deepening Search (IDS)

Limit = 2



# Today: Practical Implementation

- Practical implementation of AI problem solving as search
  - Introduced the python code for search last week, only spent last few minutes on that
  - Explore the code in-class and try to solve the activity together

# Analysis of BFS

# Properties of Breadth-First Search (BFS)

- **Completeness:**
  - a search algorithm is **complete** if it finds a solution whenever one exists
- **Optimality:**
  - a search algorithm is **optimal** if it returns a minimum-cost path if a solution exists
- **Time complexity:**
  - time required by the algorithm
- **Space complexity:**
  - memory required by the algorithm

# Properties of Breadth-First Search (BFS)

- Is BFS Complete?
- Is BFS Optimal?
- What is the running time complexity of BFS?
- What are the memory requirement of BFS?

# Properties of Breadth-First Search (BFS)

**b**: branching factor

**d**: depth of shallowest goal node

- Is BFS complete?
  - BFS is complete for a finite branching factor **b**
- Is BFS optimal?
  - BFS is optimal if step cost is 1
  - BFS will find the goal node at the shallowest depth before reaching goal nodes at a deeper level

# Let's Analyze 8-puzzle with BFS

**Initial state**

|   |   |   |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 0 | 7 | 8 |



**Goal state**

|   |   |   |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 0 |
| 7 | 8 | 0 |

# Let's Analyze 8-puzzle with BFS

Initial state

|   |   |   |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 0 | 7 | 8 |

Goal state

|   |   |   |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 0 |
| 7 | 8 | 6 |

# Let's Analyze 8-puzzle with BFS

Initial state

|   |   |   |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 0 | 7 | 8 |

Goal state

|   |   |   |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 0 |
| 7 | 8 | 0 |



# Let's Analyze 8-puzzle with BFS

Initial state

|   |   |   |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 0 | 7 | 8 |

Goal state

|   |   |   |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 0 |
| 7 | 8 |   |

UP

RIGHT

|   |   |   |
|---|---|---|
| 1 | 2 | 3 |
| 0 | 5 | 6 |
| 4 | 7 | 8 |

|   |   |   |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 0 | 8 |

UP

DOWN

RIGHT

|   |   |   |
|---|---|---|
| 0 | 2 | 3 |
| 1 | 5 | 6 |
| 4 | 7 | 8 |

|   |   |   |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 0 | 7 | 8 |

|   |   |   |
|---|---|---|
| 1 | 2 | 3 |
| 5 | 0 | 6 |
| 4 | 7 | 8 |

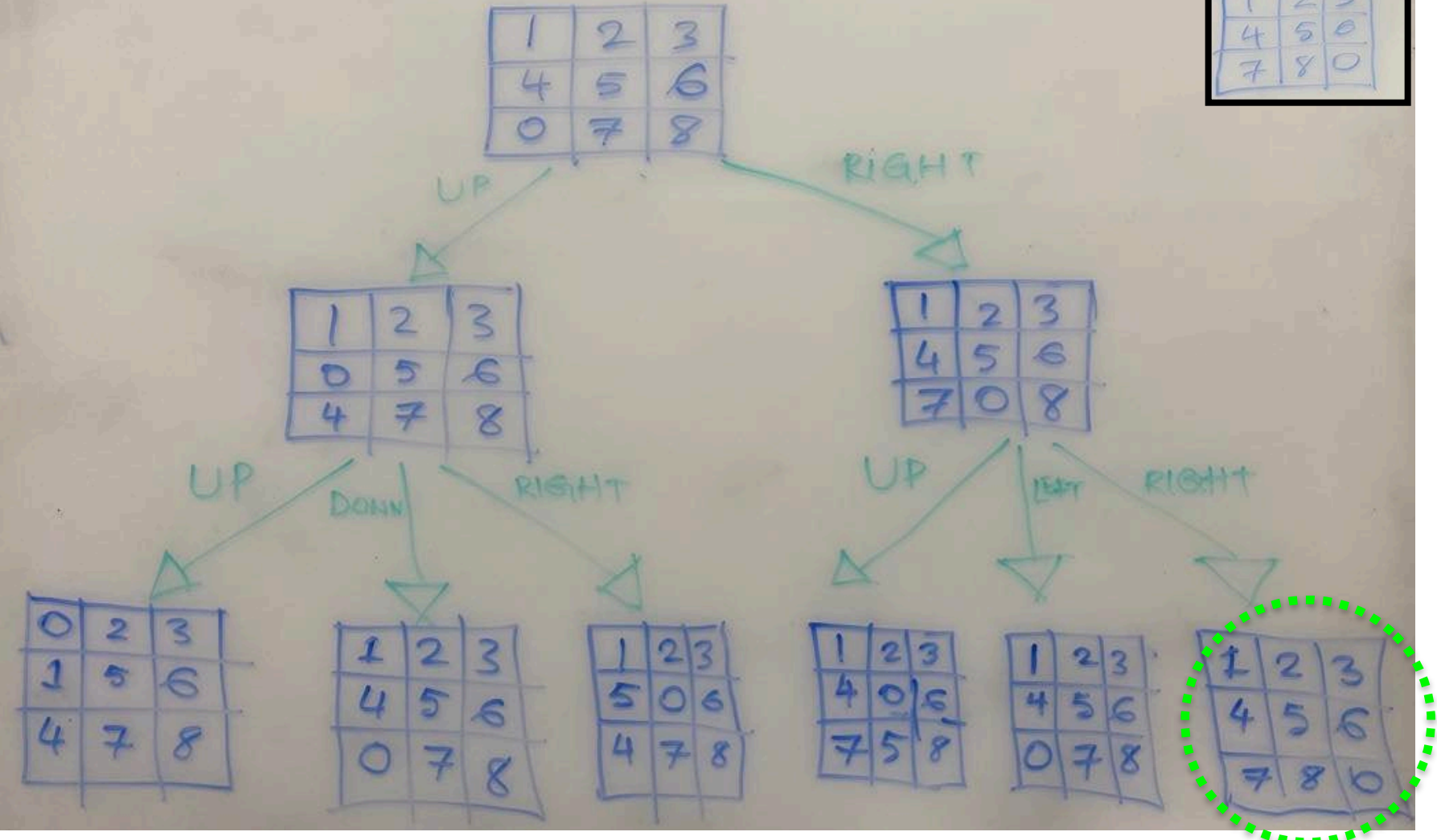
# Let's Analyze 8-puzzle with BFS

Initial state

|   |   |   |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 0 | 7 | 8 |

Goal state

|   |   |   |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 0 |
| 7 | 8 | 6 |



# Properties of Breadth-First Search (BFS)

**b**: branching factor

**d**: depth of shallowest goal node

- Time complexity of BFS?

Number of nodes generated (in the worst-case):

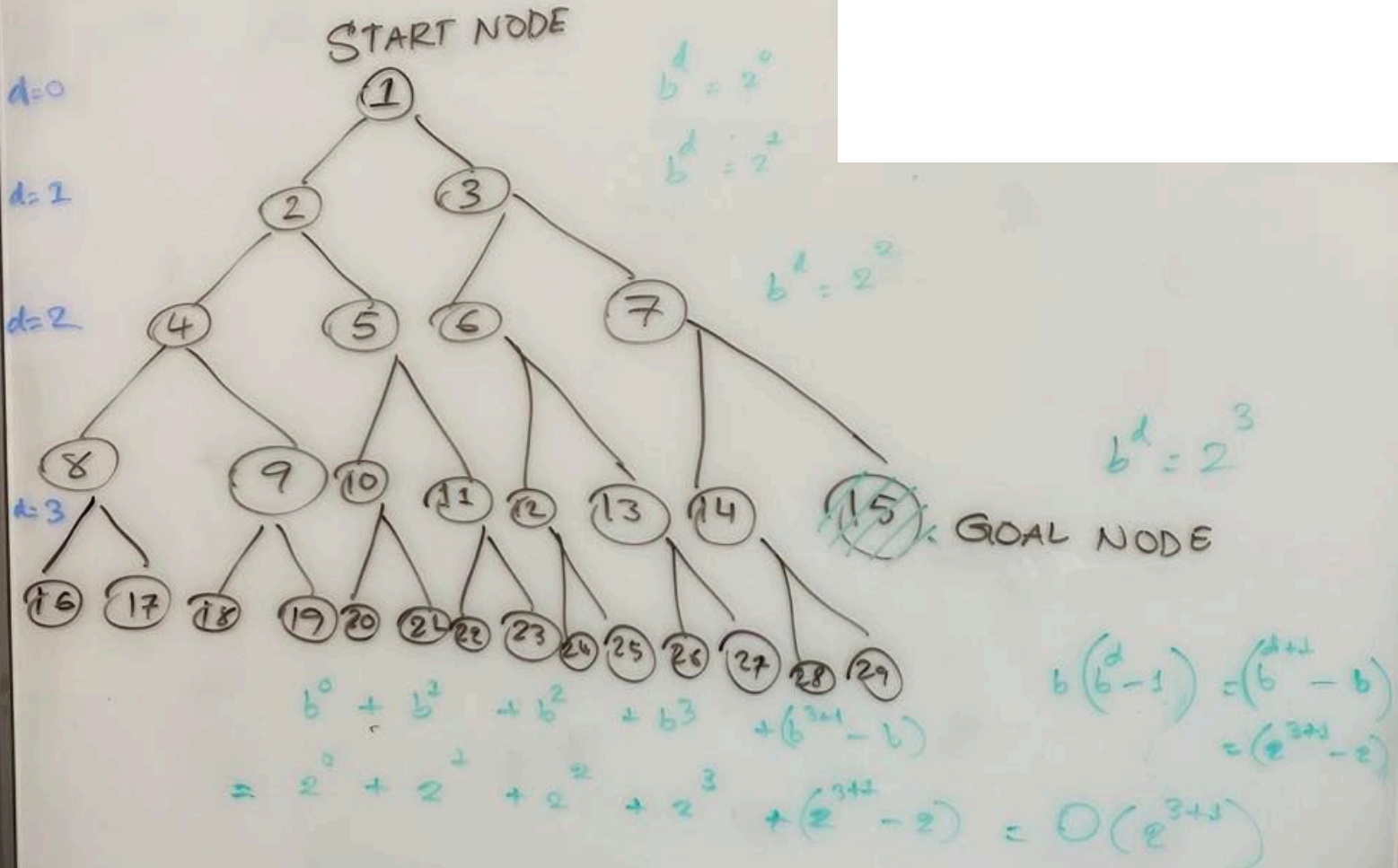
$$1 + b + b^2 + \dots + b^d + b*(b^d - 1) = O(b^{d+1})$$

Time complexity is  $O(b^{d+1})$

- Let's check — how did we arrive at above formula.

$b = 2$  : BRANCHING FACTOR  
 $d = 3$  : GOAL NODE AT THE SHALLOWEST DEPTH

## TIME COMPLEXITY



# Properties of Breadth-First Search (BFS)

- Time is a big concern for BFS
  - If the problem has a solution at depth 16 with branching factor 10, it may take about 350 years for BFS to find it

| <b>d</b>  | <b># Nodes</b> | <b>Time</b> |
|-----------|----------------|-------------|
| <b>2</b>  | 111            | 0.11 msec   |
| <b>4</b>  | 11,111         | 11.11 msec  |
| <b>6</b>  | $\sim 10^6$    | 1 sec       |
| <b>8</b>  | $\sim 10^8$    | 100 sec     |
| <b>10</b> | $\sim 10^{10}$ | 2.8 hours   |
| <b>12</b> | $\sim 10^{12}$ | 11.6 days   |
| <b>14</b> | $\sim 10^{14}$ | 3.2 years   |

## Assumptions:

- $b = 10$
- A computer can process 1,000,000 nodes/sec

# Properties of Breadth-First Search (BFS)

**b**: branching factor

**d**: depth of shallowest goal node

- Memory complexity of BFS?

Number of nodes kept in the frontier

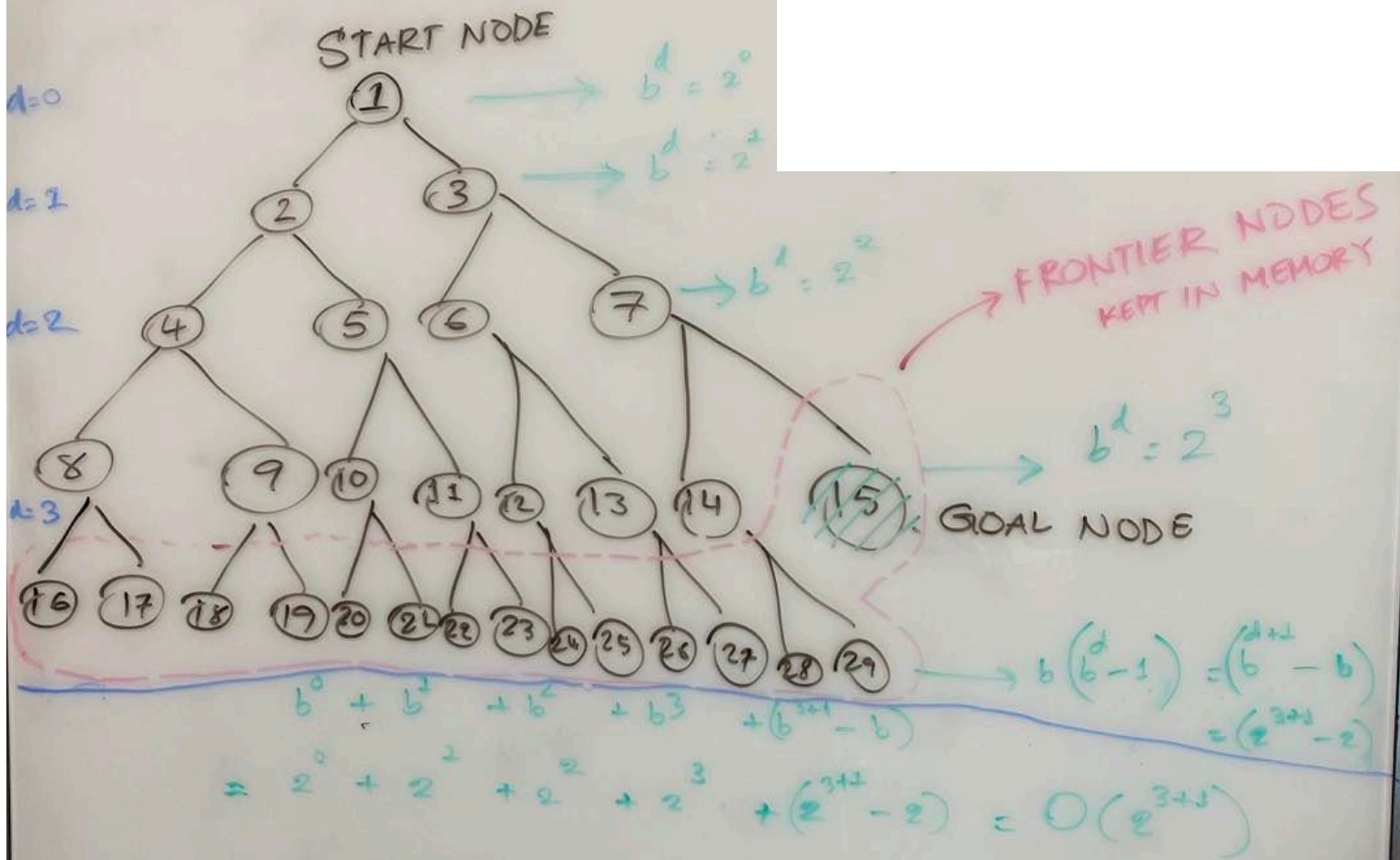
$$b * (b^d - 1) = O(b^{d+1})$$

Time and space complexity is  $O(b^{d+1})$

- Memory is a big concern for BFS.
  - can easily generate nodes at 100MB/second so gigabytes of space required if BFS runs for hours

$b = 2$  : BRANCHING FACTOR  
 $d = 3$  : GOAL NODE AT THE SHALLOWEST DEPTH

TIME COMPLEXITY



# Properties of Breadth-First Search (BFS)

- Time is a big concern for BFS
  - If the problem has a solution at depth 16 with branching factor 10, it may take about 350 years for BFS to find it

| <b>d</b>  | <b># Nodes</b> | <b>Time</b> | <b>Memory</b> |
|-----------|----------------|-------------|---------------|
| <b>2</b>  | 111            | 0.11 msec   | 11 Kbytes     |
| <b>4</b>  | 11,111         | 11.11 msec  | 1 Mbyte       |
| <b>6</b>  | $\sim 10^6$    | 1 sec       | 100 Mb        |
| <b>8</b>  | $\sim 10^8$    | 100 sec     | 10 Gbytes     |
| <b>10</b> | $\sim 10^{10}$ | 2.8 hours   | 1 Tbyte       |
| <b>12</b> | $\sim 10^{12}$ | 11.6 days   | 100 Tbytes    |
| <b>14</b> | $\sim 10^{14}$ | 3.2 years   | 10,000 Tbytes |

## Assumptions:

- $b = 10$
- A computer can process 1,000,000 nodes/sec

# Analysis of DFS

# Properties of Depth-First Search

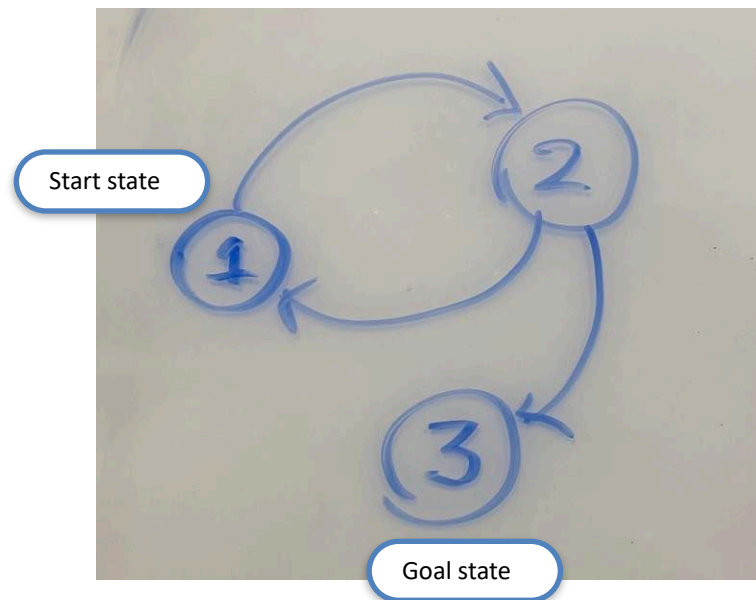
- **Completeness:**
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- **Time complexity:**
  - time required by the algorithm
- **Space complexity:**
  - memory required by the algorithm

# Properties of Depth-First Search

- Is DFS Complete?
- Is DFS Optimal?
- What is the running time complexity of DFS?
- What are the memory requirement of DFS?

# Properties of Depth-First Search

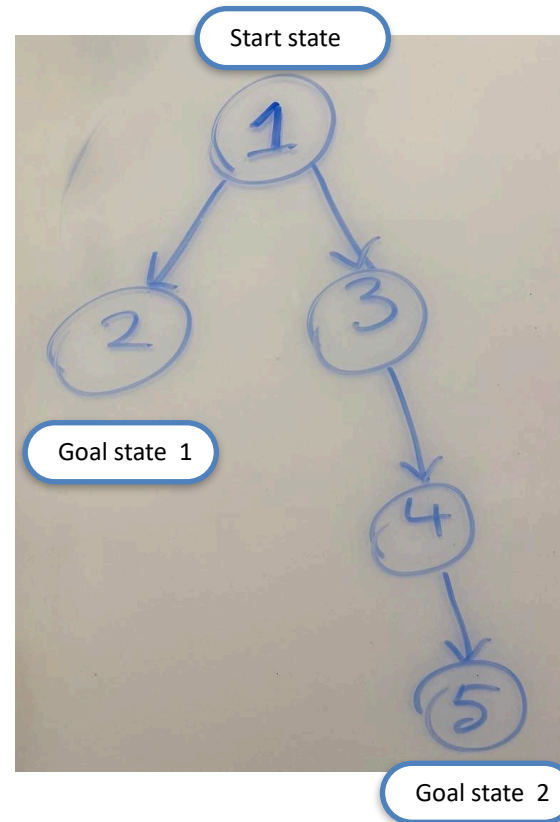
- Is DFS Complete?



- It is **not complete** even if the state graph is finite
  - The search tree could be infinite
  - Eg,  $[1] \rightarrow [2] \rightarrow [3, 1] \rightarrow [3, 2] \rightarrow [3, 3, 1] \rightarrow [3, 3, 2] \rightarrow \dots$

# Properties of Depth-First Search

- Is DFS Optimal?



- It is **not optimal**
  - The search tree could not reach goal state at a lower depth
  - Eg,  $[1] \rightarrow [2,3] \rightarrow [2,4] \rightarrow [2,5]$

# Properties of Depth-First Search

$b$ : branching factor

$l$ : depth of deepest leaf node

- Depth-first search
  - Not complete
  - Not optimal
- Number of nodes generated:  
 $1 + b + b^2 + \dots + b^l = O(b^l)$
- Time and space complexity is  $O(b \cdot l)$

# Analysis of IDS

# Properties of Iterative Deepening Search (IDS)

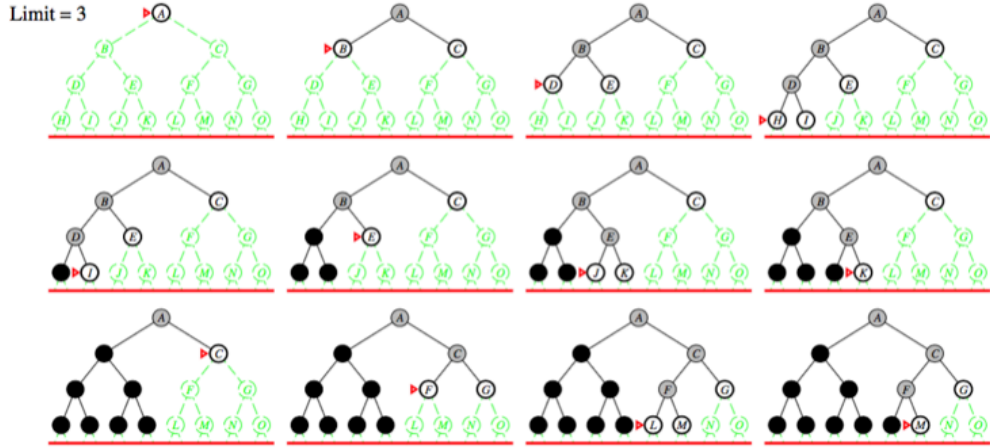
- Is IDS Complete?
- What is the running time complexity of IDS?
- What are the memory requirement of IDS?
- Is IDS Optimal?

# Properties of Iterative Deepening Search (IDS)

- IDS is complete
- IDS is optimal
- Number of nodes generated:

$$(d+1)b^0 + db^1 + (d-1)b^2 + \dots + b^d = O(b^d)$$

- Space complexity is  $O(b*d)$



# Number of Generated Nodes: BFS vs IDS

**b**: branching factor

**d**: depth of shallowest goal node

- **b** = 2 and **d** = 5

|                  | BFS       | IDS                |
|------------------|-----------|--------------------|
| Nodes at depth 1 | 1         | $1 \times 6 = 6$   |
| Nodes at depth 2 | 2         | $2 \times 5 = 10$  |
| Nodes at depth 3 | 4         | $4 \times 4 = 16$  |
| Nodes at depth 4 | 8         | $8 \times 3 = 24$  |
| Nodes at depth 5 | 16        | $16 \times 2 = 32$ |
| Nodes at depth 5 | 32        | $32 \times 1 = 32$ |
|                  | <b>63</b> | <b>120</b>         |

# Number of Generated Nodes: BFS vs IDS

**b**: branching factor

**d**: depth of shallowest goal node

- **b** = 10 and **d** = 5

| BFS            | IDS            |
|----------------|----------------|
| 1              | 6              |
| 10             | 50             |
| 100            | 400            |
| 1,000          | 3,000          |
| 10,000         | 20,000         |
| 100,000        | 100,000        |
| <b>111,111</b> | <b>123,456</b> |

# Project Assignment#2

- Implement BFS, DFS, and IDS on Mapbot
  - base code will be given; modify and append additional python snippets
  - answer questions
- Will be released later tonight!