CS143: Artificial Intelligence

Applications of Bayes' Rule

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Final Exam Announcement!

CS143 final exam will take place here in this room. Below are the other informations:

Wednesday on **5/13/2026**

12:00 pm - 1:50 pm (**1.83 hours**)

Science Connector Building (SCB#101)

Bayes' Rule

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)} = \frac{P(X|Y)P(Y)}{\sum_y P(X, Y = y)} = \frac{P(X|Y)P(Y)}{\sum_y P(X|Y = y)P(Y = y)}$$

Bayes' rule is useful when you want to know something about ***Y***, but all you can *directly* observe is ***X***

- This process is called Bayesian inference

Bayes' Rule

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)} = \frac{P(X|Y)P(Y)}{\sum_y P(X, Y = y)} = \frac{P(X|Y)P(Y)}{\sum_y P(X|Y = y)P(Y = y)}$$

$P(Y)$ is called the prior probability

It represents what we know about **y** before we consider **x**

$P(X|Y)$ is called the likelihood

We will often define the relationship between **y** and **x** in terms of conditional probability

$P(X)$ is called the evidence

$P(Y|X)$ is called the posterior probability

It represents what we know about **y** given **x**

If we know x has a distribution p(x) then p(y|x) is a more informative distribution

Bayes' Rule

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)} = \frac{P(X|Y)P(Y)}{\sum_y P(X, Y = y)} = \frac{P(X|Y)P(Y)}{\sum_y P(X|Y = y)P(Y = y)}$$

$P(Y)$ is called the prior probability

When an agent starts to build a probabilistic model, this quantity represents its initial belief by taking all background information into account

$P(X|Y)$ is called the likelihood

We will often define the relationship between **y** and **x** in terms of conditional probability

$P(X)$ is called the evidence

$P(Y|X)$ is called the posterior probability

This quantity specifies how to revise agent's belief based on new information.

Bayes' Rule

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)}$$

Bayes' rule is often useful for reasoning about causal relationships.

Disease prediction:

We have knowledge of symptoms when a disease is present: P(symptoms | disease)

In the future, if symptoms happens, then would like to reason: P(disease | symptoms)

Bayes' Rule

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)}$$

Bayes' rule is often useful for reasoning about causal relationships.

Fire prediction:

We have knowledge of alarm when a fire is present: P(alarm | fire)

In the future, if alarm happens, then would like to reason: P(fire | alarm)

Bayes' Rule Exercise#1

Accident Prediction

Exercise # 1

A cab was involved in a hit-and-run accident at night. Two cab companies, the Green and the Blue, operate in the city. You are given the following data:

- **85%** of the cabs in the city are Green and **15%** are Blue
- A witness identified the cab as Blue. The court tested the reliability of the witness in the circumstances that existed on the night of the accident and concluded that the witness correctly identifies each one of the two colors **80%** of the time and failed **20%** of the time.

What is the probability that the cab involved in the accident was Blue?

Exercise # 1

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)} = \frac{P(X|Y)P(Y)}{\sum_y P(X, Y = y)} = \frac{P(X|Y)P(Y)}{\sum_y P(X|Y = y)P(Y = y)}$$

What are the random variables?

Cab: **Y** and its sample space = {`green`, `blue`}

Witness: **X** and its sample space = {`saysgreen`, `saysblue`}

How can you express our question in terms of probabilistic causal reasoning?

$$P(Y = blue \,|\, X = saysblue) = ?$$

Using Bayes' rule

$$= \frac{P(X = saysblue \,|\, Y = blue)P(Y = blue)}{P(X = saysblue)}$$

Compute marginal distribution

Exercise # 1

What are the random variables?

Cab: **Y** and its sample space = {`green`, `blue`}

Witness: **X** and its sample space = {`saysgreen`, `saysblue`}

Express the facts from the given information table in terms of probabilities.

A cab was involved in a hit-and-run accident at night. Two cab companies, the Green and the Blue, operate in the city. You are given the following data:

- **85%** of the cabs in the city are Green and **15%** are Blue
- A witness identified the cab as Blue. The court tested the reliability of the witness in the circumstances that existed on the night of the accident and concluded that the witness correctly identifies each one of the two colors **80%** of the time and failed **20%** of the time.

$$P(Y = green) = 0.85$$

Exercise # 1

What are the random variables?

Cab: **Y** and its sample space = {`green`, `blue`}

Witness: **X** and its sample space = {`saysgreen`, `saysblue`}

Express the facts from the given information table in terms of probabilities.

A cab was involved in a hit-and-run accident at night. Two cab companies, the Green and the Blue, operate in the city. You are given the following data:

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- A witness identified the cab as Blue. The court tested the reliability of the witness in the circumstances that existed on the night of the accident and concluded that the witness correctly identifies each one of the two colors **80%** of the time and failed **20%** of the time.

$$P(Y = green) = 0.85$$
$$P(Y = blue) = 0.15$$

Exercise # 1

What are the random variables?

Cab: **Y** and its sample space = {`green`, `blue`}

Witness: **X** and its sample space = {`saysgreen`, `saysblue`}

Express the facts from the given information table in terms of probabilities.

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$$P(Y = green) = 0.85$$

$$P(Y = blue) = 0.15$$

$$P(X = saysblue \,|\, Y = blue) = 0.80$$

Exercise # 1

What are the random variables?

Cab: **Y** and its sample space = {`green, blue`}

Witness: **X** and its sample space = {`saysgreen, saysblue`}

Express the facts from the given information table in terms of probabilities.

> A cab was involved in a hit-and-run accident at night. Two cab companies, the Green and the Blue, operate in the city. You are given the following data:
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> - **85%** of the cabs in the city are Green and **15%** are Blue
> - A witness identified the cab as Blue. The court tested the reliability of the witness in the circumstances that existed on the night of the accident and concluded that the witness correctly identifies each one of the two colors **80%** of the time and failed **20%** of the time.

$$P(Y = green) = 0.85$$
$$P(Y = blue) = 0.15$$

$$P(X = saysblue \mid Y = blue) = 0.80$$
$$P(X = saysgreen \mid Y = blue) = 0.20$$

Exercise # 1

What are the random variables?

Cab: **Y** and its sample space = {`green`, `blue`}

Witness: **X** and its sample space = {`saysgreen`, `saysblue`}

Express the facts from the given information table in terms of probabilities.

A cab was involved in a hit-and-run accident at night. Two cab companies, the Green and the Blue, operate in the city. You are given the following data:

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- A witness identified the cab as Blue. The court tested the reliability of the witness in the circumstances that existed on the night of the accident and concluded that the witness correctly identifies each one of the two colors **80%** of the time and failed **20%** of the time.

$$P(Y = green) = 0.85$$
$$P(Y = blue) = 0.15$$

$$P(X = saysblue \,|\, Y = blue) = 0.80$$
$$P(X = saysgreen \,|\, Y = blue) = 0.20$$

$$P(X = saysgreen \,|\, Y = green) = 0.80$$

Exercise # 1

What are the random variables?

Cab: **Y** and its sample space = {`green`, `blue`}

Witness: **X** and its sample space = {`saysgreen`, `saysblue`}

Express the facts from the given information table in terms of probabilities.

A cab was involved in a hit-and-run accident at night. Two cab companies, the Green and the Blue, operate in the city. You are given the following data:

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- A witness identified the cab as Blue. The court tested the reliability of the witness in the circumstances that existed on the night of the accident and concluded that the witness correctly identifies each one of the two colors **80%** of the time and failed **20%** of the time.

$$P(Y = green) = 0.85$$
$$P(Y = blue) = 0.15$$

$$P(X = saysblue \,|\, Y = blue) = 0.80$$
$$P(X = saysgreen \,|\, Y = blue) = 0.20$$

$$P(X = saysgreen \,|\, Y = green) = 0.80$$
$$P(X = saysblue \,|\, Y = green) = 0.20$$

Exercise # 1

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)} = \frac{P(X|Y)P(Y)}{\sum_y P(X, Y=y)} = \frac{P(X|Y)P(Y)}{\sum_y P(X|Y=y)P(Y=y)}$$

$$P(Y = green) = 0.85$$
$$P(Y = blue) = 0.15$$

$$P(X = saysblue \,|\, Y = blue) = 0.80$$
$$P(X = saysgreen \,|\, Y = blue) = 0.20$$

$$P(X = saysgreen \,|\, Y = green) = 0.80$$
$$P(X = saysblue \,|\, Y = green) = 0.20$$

$$P(Y = blue \,|\, X = saysblue) = \frac{P(X = saysblue \,|\, Y = blue)P(Y = blue)}{P(X = saysblue)}$$

Exercise # 1

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)} = \frac{P(X|Y)P(Y)}{\sum_y P(X, Y = y)} = \frac{P(X|Y)P(Y)}{\sum_y P(X|Y = y)P(Y = y)}$$

$$P(Y = green) = 0.85$$
$$P(Y = blue) = 0.15$$

$$P(X = saysblue \,|\, Y = blue) = 0.80$$
$$P(X = saysgreen \,|\, Y = blue) = 0.20$$

$$P(X = saysgreen \,|\, Y = green) = 0.80$$
$$P(X = saysblue \,|\, Y = green) = 0.20$$

$$P(Y = blue \,|\, X = saysblue) = \frac{P(X = saysblue \,|\, Y = blue)P(Y = blue)}{P(X = saysblue)}$$
$$= \frac{0.80 * 0.15}{P(X = saysblue \,|\, Y = blue)P(Y = blue) + P(X = saysblue \,|\, Y = green)P(Y = green)}$$

Exercise # 1

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)} = \frac{P(X|Y)P(Y)}{\sum_y P(X, Y = y)} = \frac{P(X|Y)P(Y)}{\sum_y P(X|Y = y)P(Y = y)}$$

$$P(Y = green) = 0.85$$
$$P(Y = blue) = 0.15$$

$$P(X = saysblue \,|\, Y = blue) = 0.80$$
$$P(X = saysgreen \,|\, Y = blue) = 0.20$$

$$P(X = saysgreen \,|\, Y = green) = 0.80$$
$$P(X = saysblue \,|\, Y = green) = 0.20$$

$$P(Y = blue \,|\, X = saysblue) = \frac{P(X = saysblue \,|\, Y = blue)P(Y = blue)}{P(X = saysblue)}$$

$$= \frac{0.80 * 0.15}{P(X = saysblue \,|\, Y = blue)P(Y = blue) + P(X = saysblue \,|\, Y = green)P(Y = green)}$$

$$= \frac{0.80 * 0.15}{0.80 * 0.15 + P(X = correct \,|\, Y = green)P(Y = green)}$$

Exercise # 1

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)} = \frac{P(X|Y)P(Y)}{\sum_y P(X, Y = y)} = \frac{P(X|Y)P(Y)}{\sum_y P(X|Y = y)P(Y = y)}$$

$$P(Y = green) = 0.85$$
$$P(Y = blue) = 0.15$$

$$P(X = saysblue \mid Y = blue) = 0.80$$
$$P(X = saysgreen \mid Y = blue) = 0.20$$

$$P(X = saysgreen \mid Y = green) = 0.80$$
$$P(X = saysblue \mid Y = green) = 0.20$$

$$P(Y = blue \mid X = saysblue) = \frac{P(X = saysblue \mid Y = blue)P(Y = blue)}{P(X = saysblue)}$$

$$= \frac{0.80 * 0.15}{P(X = saysblue \mid Y = blue)P(Y = blue) + P(X = saysblue \mid Y = green)P(Y = green)}$$

$$= \frac{0.80 * 0.15}{0.80 * 0.15 + P(X = correct \mid Y = green)P(Y = green)}$$

$$= \frac{0.80 * 0.15}{0.80 * 0.15 + 0.2 * 0.85}$$

Exercise # 1

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)} = \frac{P(X|Y)P(Y)}{\sum_y P(X, Y=y)} = \frac{P(X|Y)P(Y)}{\sum_y P(X|Y=y)P(Y=y)}$$

$$P(Y = green) = 0.85$$
$$P(Y = blue) = 0.15$$

$$P(X = saysblue \mid Y = blue) = 0.80$$
$$P(X = saysgreen \mid Y = blue) = 0.20$$

$$P(X = saysgreen \mid Y = green) = 0.80$$
$$P(X = saysblue \mid Y = green) = 0.20$$

$$P(Y = blue \mid X = saysblue) = \frac{P(X = saysblue \mid Y = blue)P(Y = blue)}{P(X = saysblue)}$$

$$= \frac{0.80 * 0.15}{P(X = saysblue \mid Y = blue)P(Y = blue) + P(X = saysblue \mid Y = green)P(Y = green)}$$

$$= \frac{0.80 * 0.15}{0.80 * 0.15 + P(X = correct \mid Y = green)P(Y = green)}$$

$$= \frac{0.80 * 0.15}{0.80 * 0.15 + 0.2 * 0.85}$$

$$= \frac{0.12}{0.29}$$

Exercise # 1

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)} = \frac{P(X|Y)P(Y)}{\sum_y P(X, Y = y)} = \frac{P(X|Y)P(Y)}{\sum_y P(X|Y = y)P(Y = y)}$$

$$P(Y = green) = 0.85$$
$$P(Y = blue) = 0.15$$

$$P(X = saysblue \mid Y = blue) = 0.80$$
$$P(X = saysgreen \mid Y = blue) = 0.20$$

$$P(X = saysgreen \mid Y = green) = 0.80$$
$$P(X = saysblue \mid Y = green) = 0.20$$

$$P(Y = blue \mid X = saysblue) = \frac{P(X = saysblue \mid Y = blue)P(Y = blue)}{P(X = saysblue)}$$
$$= \frac{0.80 * 0.15}{P(X = saysblue \mid Y = blue)P(Y = blue) + P(X = saysblue \mid Y = green)P(Y = green)}$$
$$= \frac{0.80 * 0.15}{0.80 * 0.15 + P(X = correct \mid Y = green)P(Y = green)}$$
$$= \frac{0.80 * 0.15}{0.80 * 0.15 + 0.2 * 0.85}$$
$$= \frac{0.12}{0.29}$$
$$= 0.41$$

Bayes' Rule Exercise#2

Coin Prediction

Exercise # 2

A magician is performing a show in Times Square. He has two coins: one fair and one biased. You are given the following information:

- He tells us in advance that he selects a coin such that, **75%** of the time, it is fair, and the other **25%** of the time, it is biased.
- The magician flips a coin, and we all observe a head. The fair coin has one head and one tail. Hence, if the selected coin is fair, it will land heads **50%** of the time and tails **50%** of the time. The biased coin has heads on both sides *(ie, two heads and zero tail)*. Hence, if the selected coin is biased, it will land heads **100%** of the time.

What is the probability that it is the fair coin?

Exercise # 2

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)} = \frac{P(X|Y)P(Y)}{\sum_y P(X, Y = y)} = \frac{P(X|Y)P(Y)}{\sum_y P(X|Y = y)P(Y = y)}$$

What are the random variables?

_______: **Y** and its sample space = {? ? ? ,   ? ? ?}

_______: **X** and its sample space = {? ? ? ,   ? ? ?}

How can you express our question in terms of probabilistic causal reasoning?

Using Bayes' rule

$$P(Y = ?\,|\,X = ?) = ?$$

$$= \frac{P(X = ?\,|\,Y = ?)P(Y = ?)}{P(X = ?)}$$

Compute marginal distribution

Exercise # 2

What are the random variables?

_______: **Y** and its sample space = {???, ???}

_______: **X** and its sample space = {???, ???}

Express the facts from the given information table in terms of probabilities.

A magician is performing a show in Times Square. He has two coins: one fair and one biased. You are given the following information:

- He tells us in advance that he selects a coin such that, **75%** of the time, it is fair, and the other **25%** of the time, it is biased.
- The magician flips a coin, and we all observe a head. The fair coin has one head and one tail. Hence, if the selected coin is fair, it will land heads **50%** of the time and tails **50%** of the time. The biased coin has heads on both sides *(ie, two heads and zero tail)*. Hence, if the selected coin is biased, it will land heads **100%** of the time.

$$P(Y = ?) = ?$$
$$P(Y = ?) = ?$$

$$P(X = ? \mid Y = ?) = ?$$
$$P(X = ? \mid Y = ?) = ?$$

$$P(X = ? \mid Y = ?) = ?$$
$$P(X = ? \mid Y = ?) = ?$$

Exercise # 2

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)} = \frac{P(X|Y)P(Y)}{\sum_y P(X, Y = y)} = \frac{P(X|Y)P(Y)}{\sum_y P(X|Y = y)P(Y = y)}$$

$$P(Y = ?) = ?$$
$$P(Y = ?) = ?$$

$$P(X = ?\,|\,Y = ?) = ?$$
$$P(X = ?\,|\,Y = ?) = ?$$

$$P(X = ?\,|\,Y = ?) = ?$$
$$P(X = ?\,|\,Y = ?) = ?$$

$$P(Y = ?\,|\,X = ?) = \frac{P(X = ?\,|\,Y = ?)P(Y = ?)}{P(X = ?)}$$
$$= \frac{?}{P(X = ?\,|\,Y = ?)P(Y = ?) + P(X = ?\,|\,Y = ?)P(Y = ?)}$$
$$= \frac{?}{? + P(X = ?\,|\,Y = ?)P(Y = ?)}$$
$$= \frac{?}{? + ?}$$
$$= \frac{?}{?}$$
$$= ?$$

Bayes' Rule Exercise#3

Disease Prediction

Exercise # 3

Prof. Reza visited Bangladesh, a country that was affected by a rare disease 100 years ago. Since so much time has passed, the likelihood of anyone being affected again is extremely low. The city's population can be divided into two groups: those who are not infected (NovidN) and those who are infected (NovidP). You are given the following information:

- **0.01%** of the population in the city is afflicted with the disease (NovidP), while the remaining **99.99%** are not (NovidN).
- Upon returning, he decided to take a test. A doctor diagnoses Prof. Reza as NovidP (positive). The hospital reports that the doctor's diagnosis is **99%** accurate, meaning the doctor correctly identifies the disease **99%** of the time and makes an error **1%** of the time.

What is the probability that Prof Reza has NovidP (positive)?

Exercise # 2

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)} = \frac{P(X|Y)P(Y)}{\sum_y P(X, Y=y)} = \frac{P(X|Y)P(Y)}{\sum_y P(X|Y=y)P(Y=y)}$$

What are the random variables?

_______: **Y** and its sample space = {???, ???}

_______: **X** and its sample space = {???, ???}

How can you express our question in terms of probabilistic causal reasoning?

$$P(Y = ?\,|\,X = ?) = ?$$

Using Bayes' rule

$$= \frac{P(X = ?\,|\,Y = ?)P(Y = ?)}{P(X = ?)}$$

Compute marginal distribution

Exercise # 2

What are the random variables?

_______: **Y** and its sample space = {???, ???}

_______: **X** and its sample space = {???, ???}

Express the facts from the given information table in terms of probabilities.

> Prof. Reza visited a country that was, at the time, affected by a rare seasonal disease. As a result, the city's population can be divided into two groups: those who are infected (NovidP) and those who are not infected (NovidN). You are given the following information:
>
> - **0.01%** of the population in the city is afflicted with the disease (NovidP), while the remaining **99.99%** are not (NovidN).
> - Upon returning, he decided to take a test. A doctor diagnoses Prof. Reza as NovidP (positive). The hospital reports that the doctor's diagnosis is **99%** accurate, meaning the doctor correctly identifies the disease **99%** of the time and makes an error **1%** of the time.

$$P(Y = ?) = ?$$
$$P(Y = ?) = ?$$

$$P(X = ? \mid Y = ?) = ?$$
$$P(X = ? \mid Y = ?) = ?$$

$$P(X = ? \mid Y = ?) = ?$$
$$P(X = ? \mid Y = ?) = ?$$

Exercise # 2

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)} = \frac{P(X|Y)P(Y)}{\sum_y P(X, Y=y)} = \frac{P(X|Y)P(Y)}{\sum_y P(X|Y=y)P(Y=y)}$$

$$P(Y=?) = ?$$
$$P(Y=?) = ?$$

$$P(X=?\,|\,Y=?) = ?$$
$$P(X=?\,|\,Y=?) = ?$$

$$P(X=?\,|\,Y=?) = ?$$
$$P(X=?\,|\,Y=?) = ?$$

$$P(Y=?\,|\,X=?) = \frac{P(X=?\,|\,Y=?)P(Y=?)}{P(X=?)}$$

$$= \frac{?}{P(X=?\,|\,Y=?)P(Y=?) + P(X=?\,|\,Y=?)P(Y=?)}$$

$$= \frac{?}{? + P(X=?\,|\,Y=?)P(Y=?)}$$

$$= \frac{?}{?+?}$$

$$= \frac{?}{?}$$

$$= ?$$