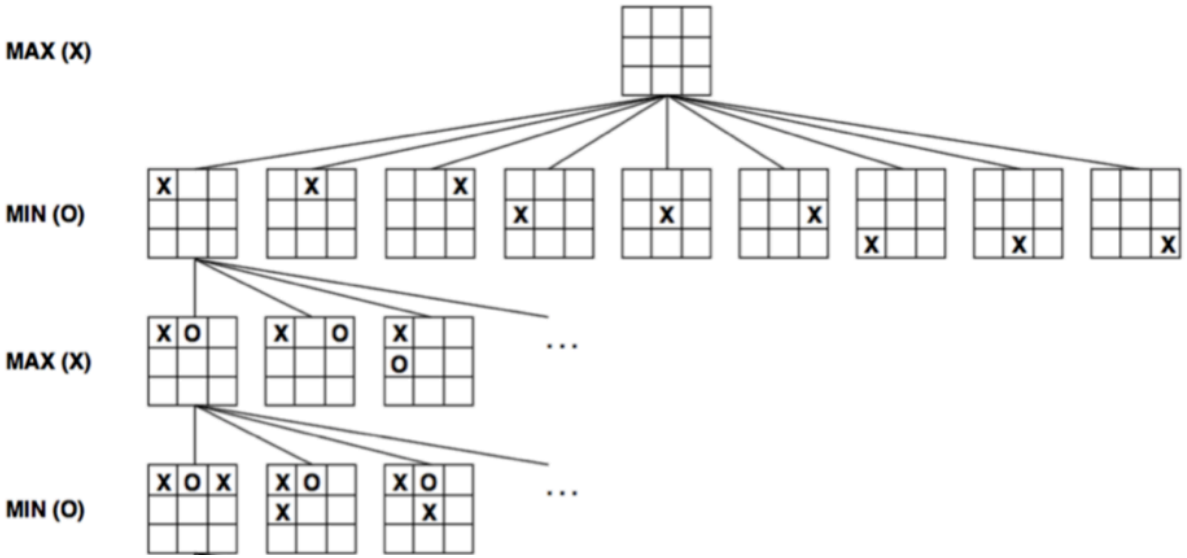
CS143: Artificial Intelligence



Adversarial Search

Drake UNIVERSITY

Adversarial Search

- "Adversarial Search" concern problems known as **games**:
  - eg, checkers, chess, go, backgammon, etc.

- We focus in particular on games that involve turn-taking and are **zero-sum**:
  - means the total payoff to players is the same for each game
  - eg, given player A vs. player B, the outcomes include:
  - **A wins (1), A loses (-1), or tie (0)**
  - A wins (1,0); B wins (0,1), or tie (0.5,0.5)

Games

- Games are interesting because they are *too hard* to solve optimally
  - eg, the search tree for a chess game involves 10^{154} possible nodes
- Consider two players: MIN and MAX
  - MAX moves first;
  - MIN moves next;
  - MAX moves again;
  - MIN follows up;
  - …
  - …
  - etc

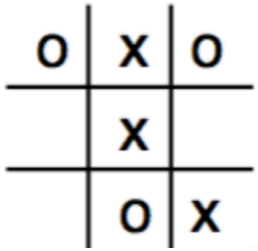
Games

- The choice of possible moves for both players gives rise to a **game tree**

- A **utility function** determines a numeric value for a game that ends in a *terminal state*

  - MAX wins (1), MIN loses (-1), or tie (0)
  - MAX wins (1,0); MIN wins (0,1), or tie (0.5,0.5)

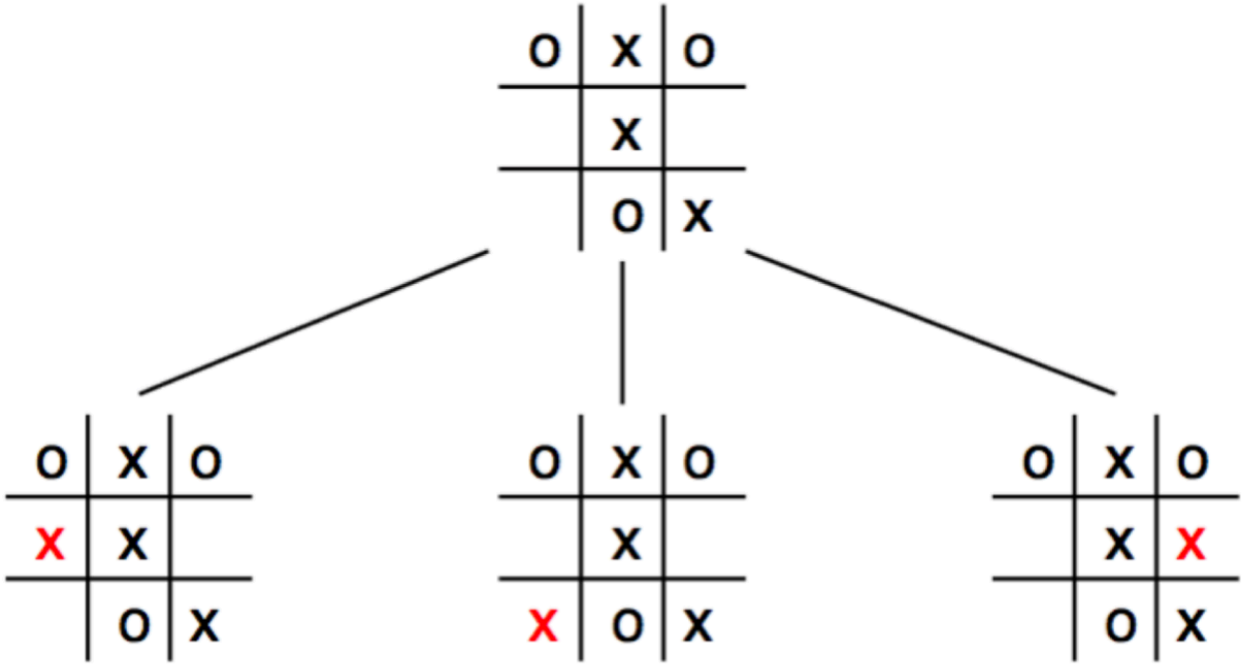
Game Tree Example



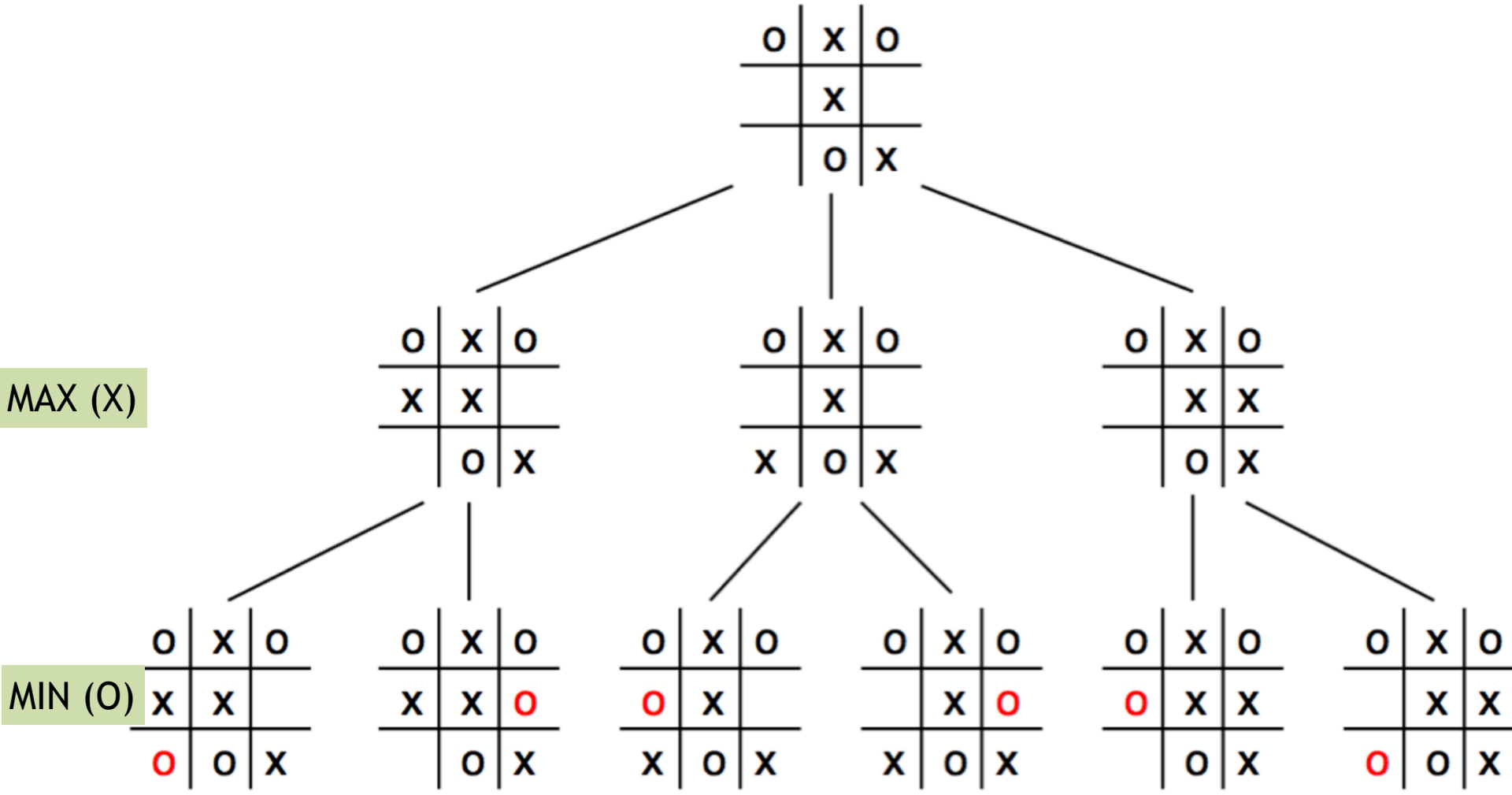
MAX (X)

Game Tree Example

MAX (X)



Game Tree Example



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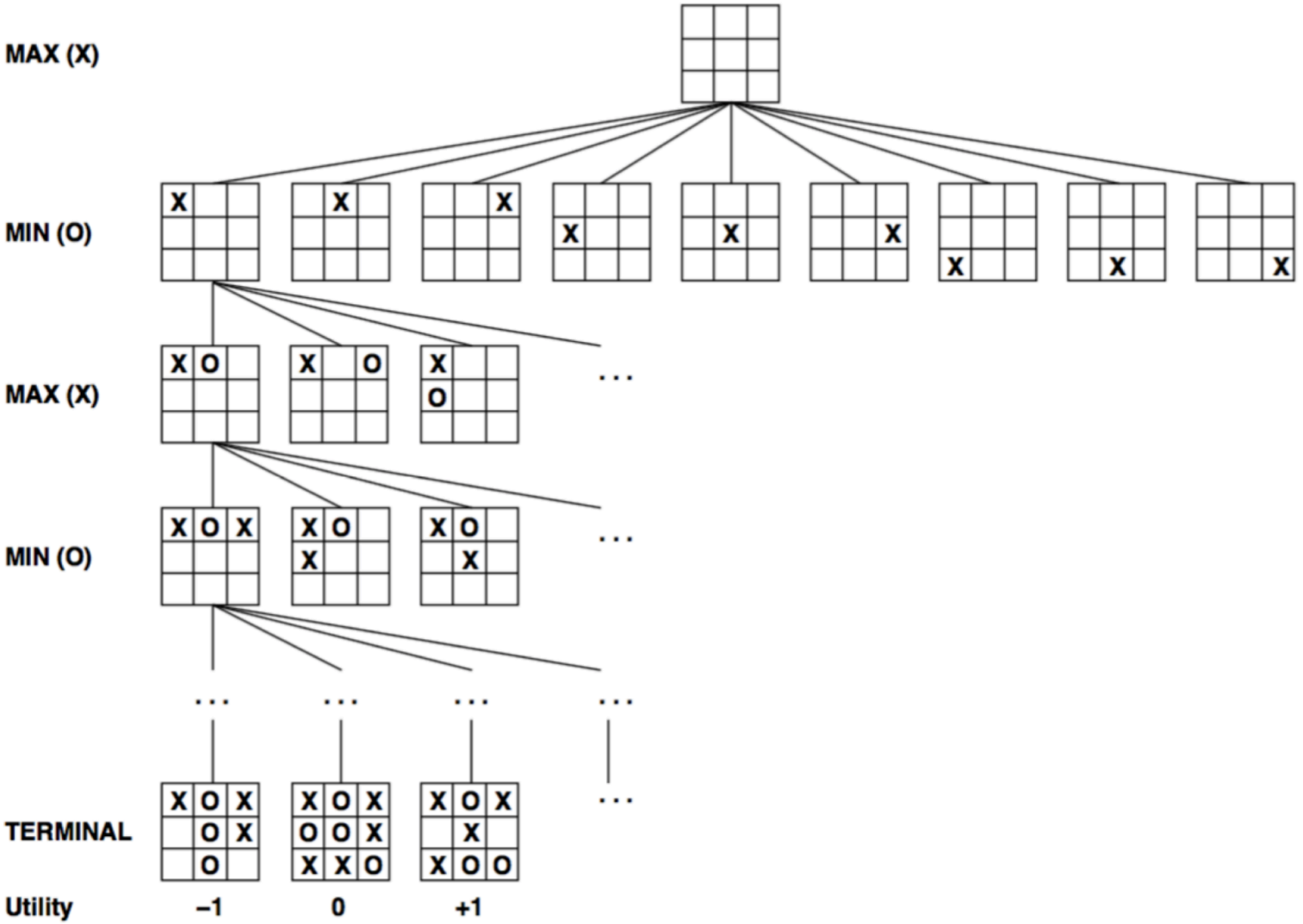


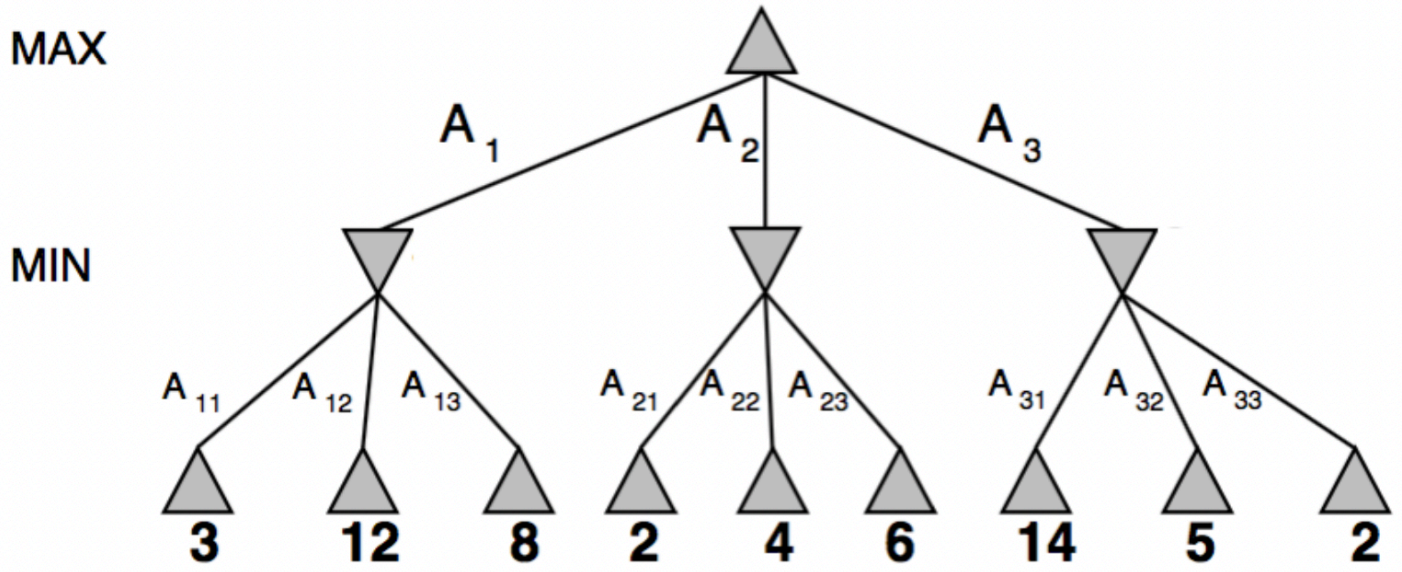
The MINIMAX algorithm

Contingent Strategies

- MAX would like to play according to a given strategy
- However, MIN has something to say about how the game unfolds
- Thus MAX must play according to a **contingent strategy**, which takes into account MIN's possible responses to MAX's moves

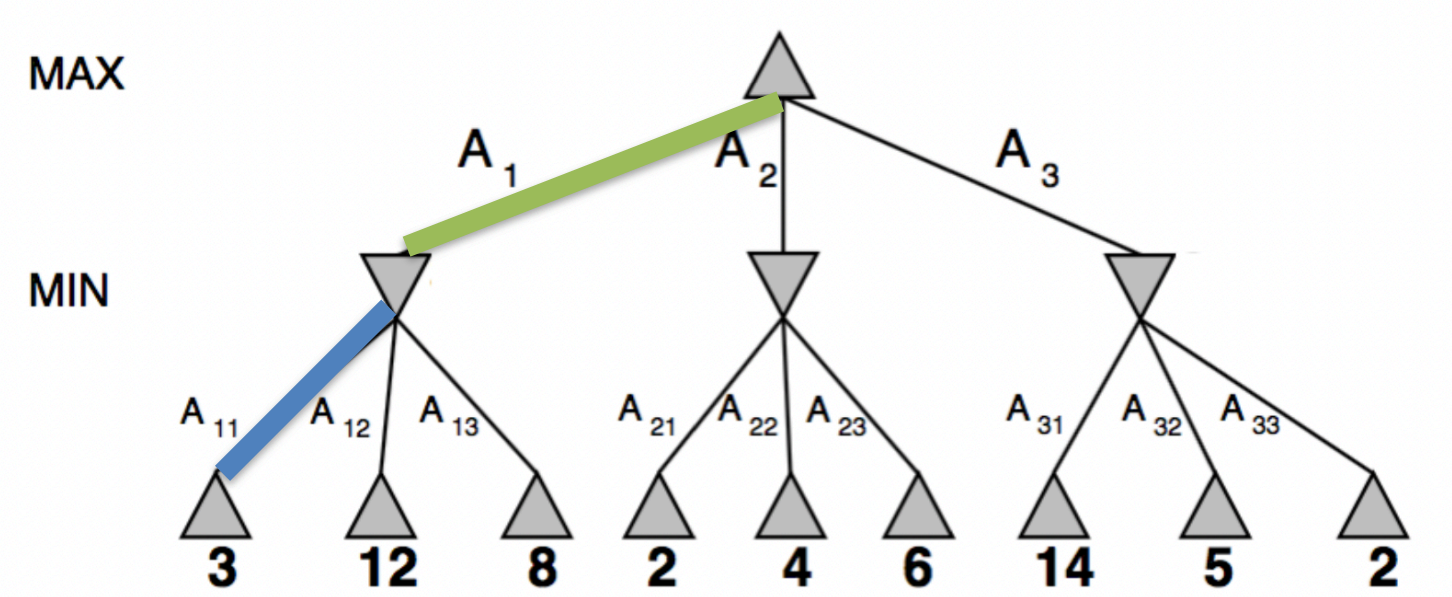
A Simple Game

- Suppose MAX and MIN play a game in which
  - MAX is trying to getting the highest possible score
  - while MIN is trying to make MAX's score as low as possible



A Simple Game

- MAX can choose among A_1, A_2, and A_3
- Once MAX makes a choice, say A_i, MIN chooses between A_{i1}, A_{i2} and A_{i3}



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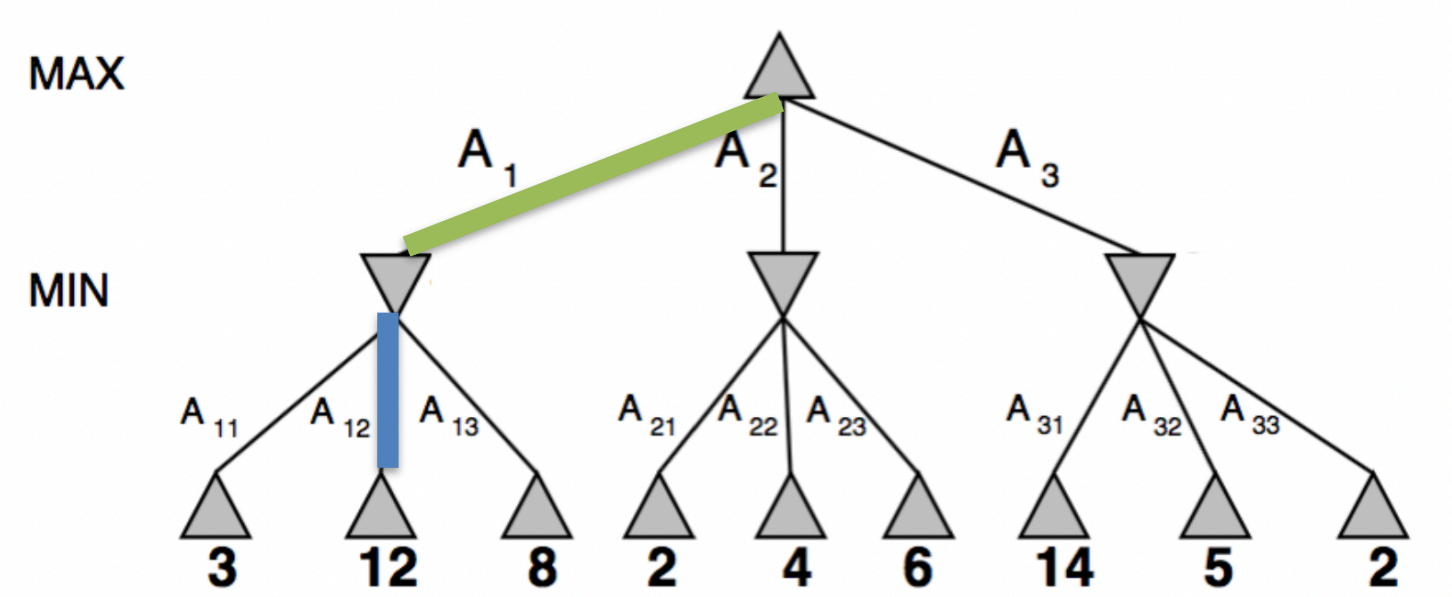

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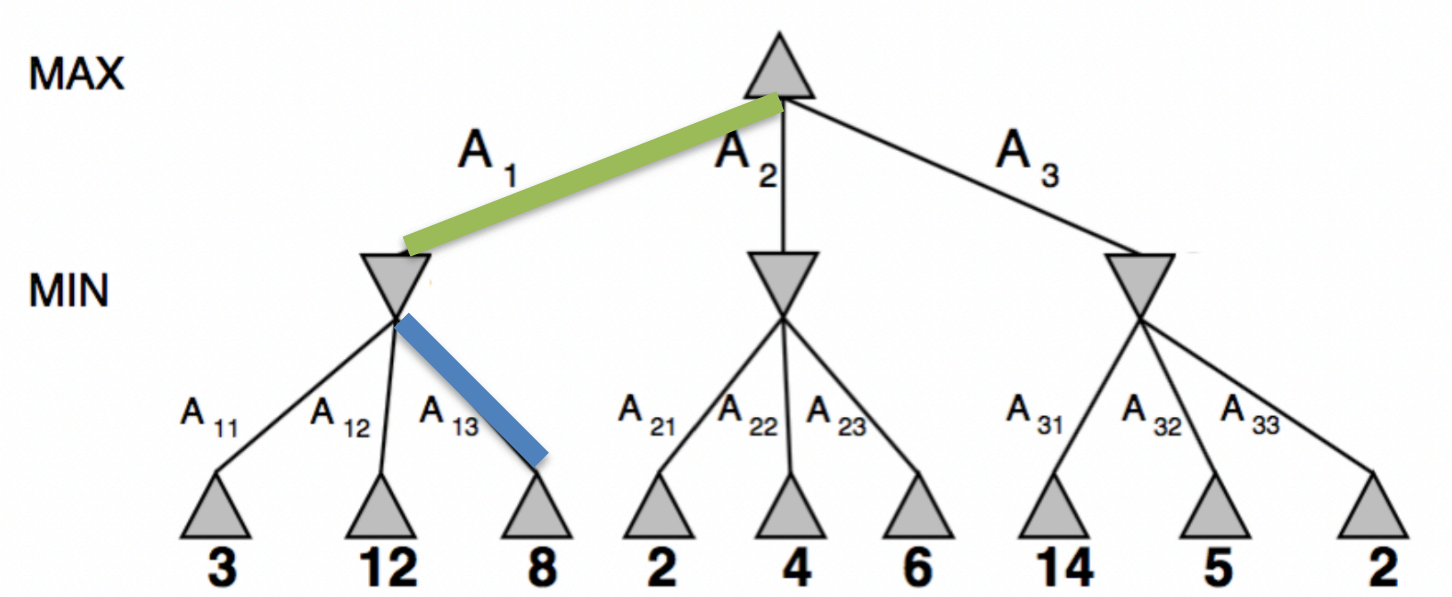
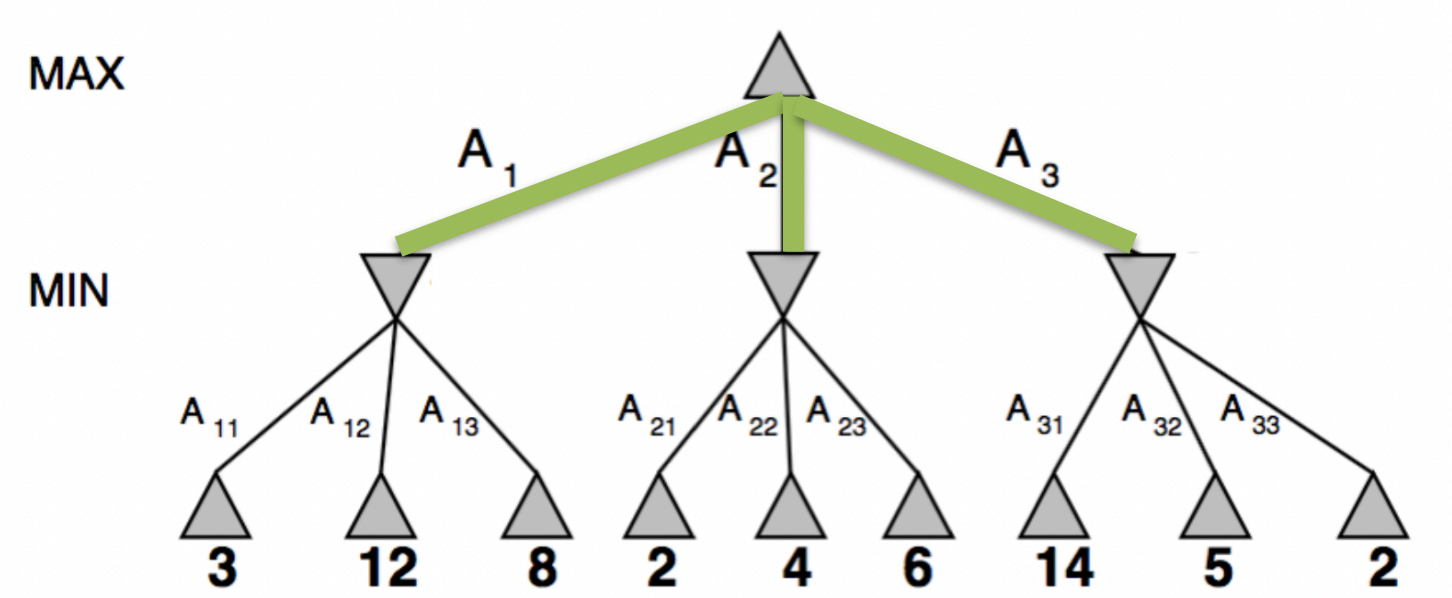


A Simple Game

- What should MAX choose (among A_1, A_2, and A_3)?



MINIMAX Algorithm

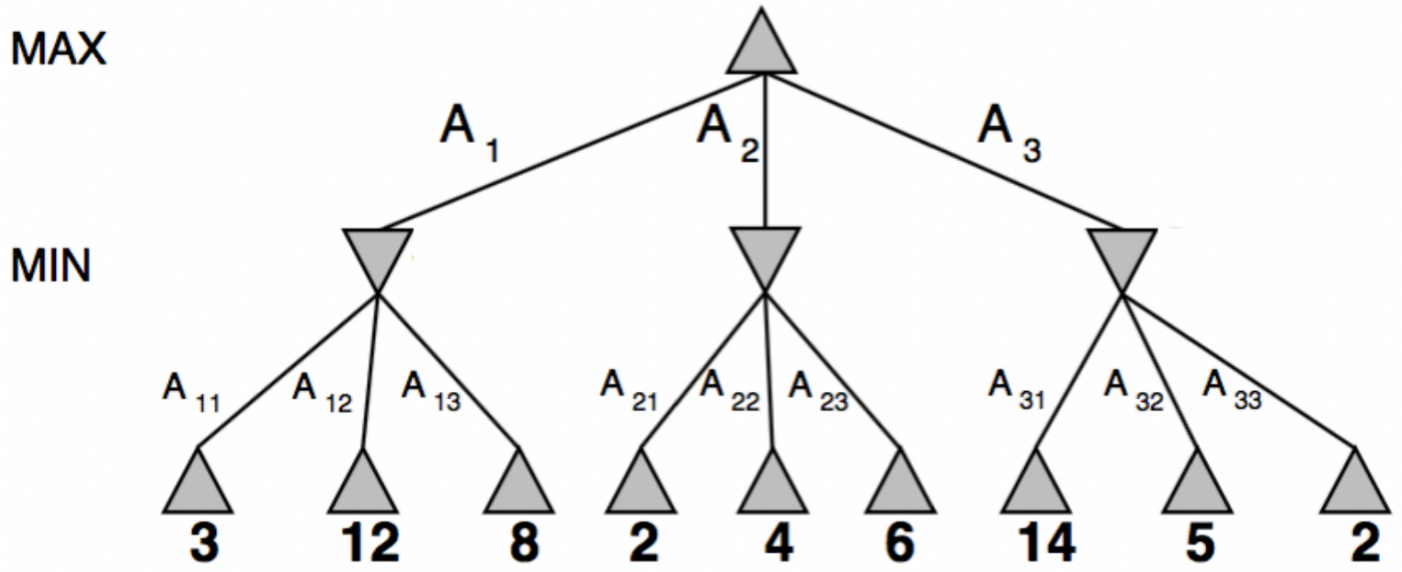
- Idea: At each turn, choose to move to the position with best **minimax** value
  - aim for the best achievable payoff against the best play of the opponent

- Evaluate the game tree's utility function value for all possible moves
  - start from the bottom-up

- This results in perfect play for deterministic, perfect-information games

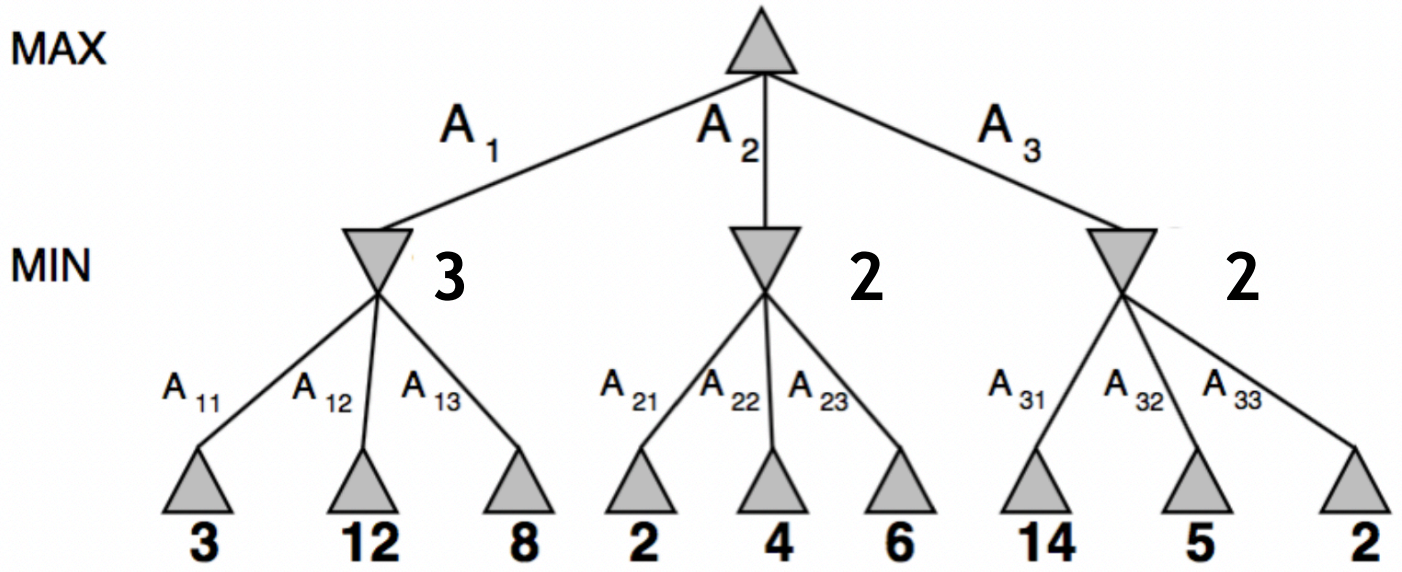
MINIMAX Algorithm

- Example: MAX and MIN play our simple game



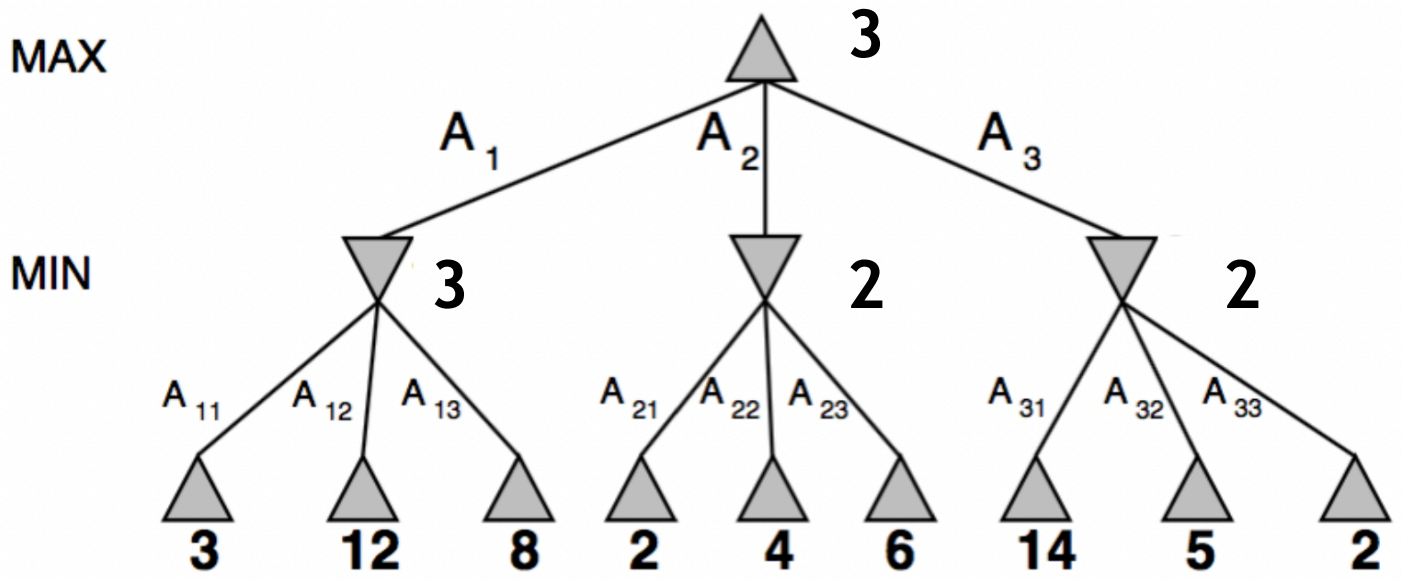
MINIMAX Algorithm

- Example: MAX and MIN play our simple game



MINIMAX Algorithm

- Example: MAX and MIN play our simple game



Assumption in MINIMAX Algorithm

- The minimax value of a node is MAX's utility of being in the corresponding state, but only assuming that both MAX and MIN play optimally from there to the end of the game
  - That is, MAX's strategy takes into account the worst-case scenario given by MIN's moves

- What happens if MIN does not play optimally?
  - One can show that MAX in fact does even better

Tile Sliding Game

- The two players, Player A and Player B, take turns
- Player A moves first

- Player B moves next

Tile Sliding Game

- During A's turn, they can do one of the followings
  - stay put
  - move right one space

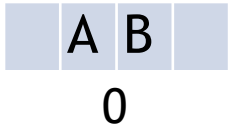
- During B's turn, they can do one of the followings
  - stay put
  - move left one space

- If the players occupy adjacent spaces, then the player may jump over the opponent to the next open space in the respective legal direction

Game Ending Conditions

- Condition 1: One of the players reaches the opposite end of the board (the player reaching the other end is the winner)
  - A wins if she reaches to the right-most position
  - B wins if he reaches to the left-most position

- Condition 2: Both players stay put in successive turns (this is a draw)



Game Ending Conditions

- Condition 3:
  The game reaches the following state:

+1

Then B stays put and A wins

or the game reaches the following state:

-1

Then A stays put and B wins

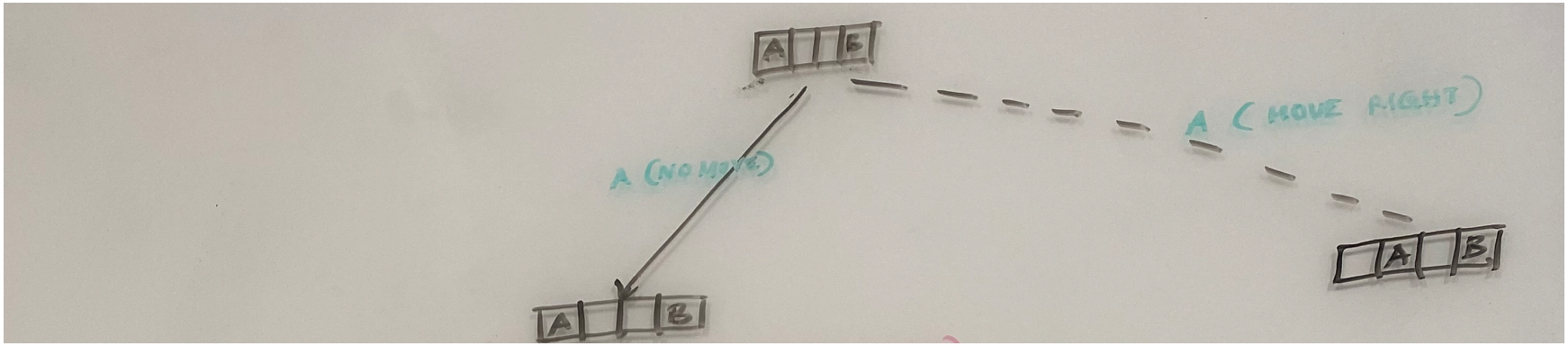
Class Activity

Tile Sliding Game Activity

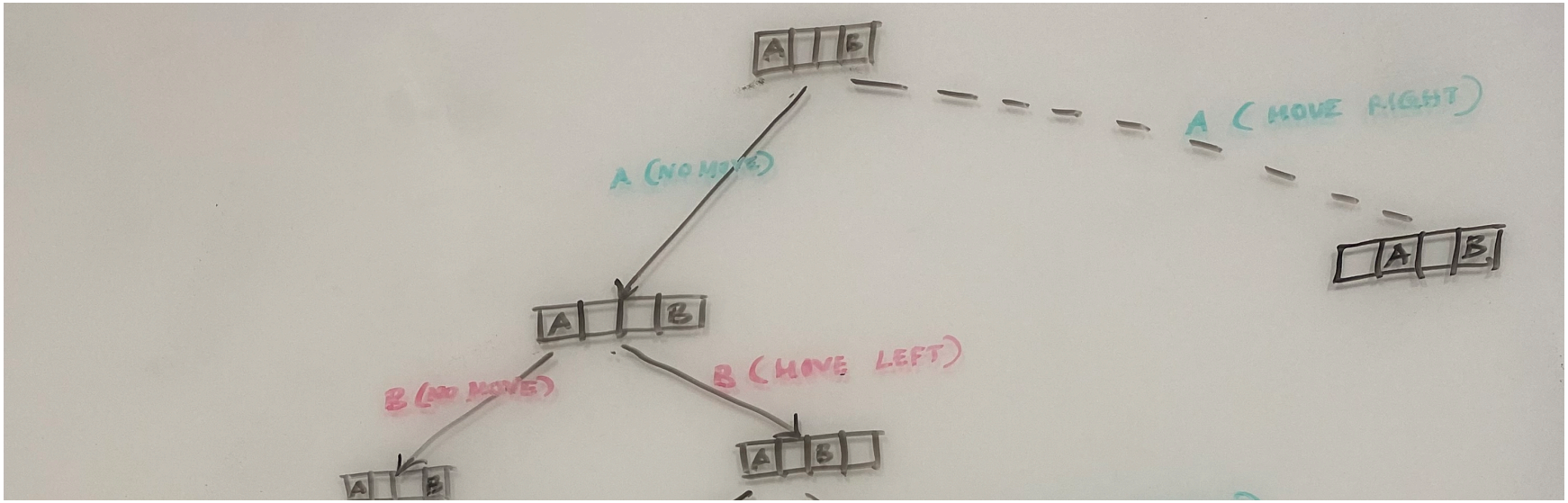
- From the above configuration, play a game with a partner and draw the game tree

Tile Sliding Game Activity

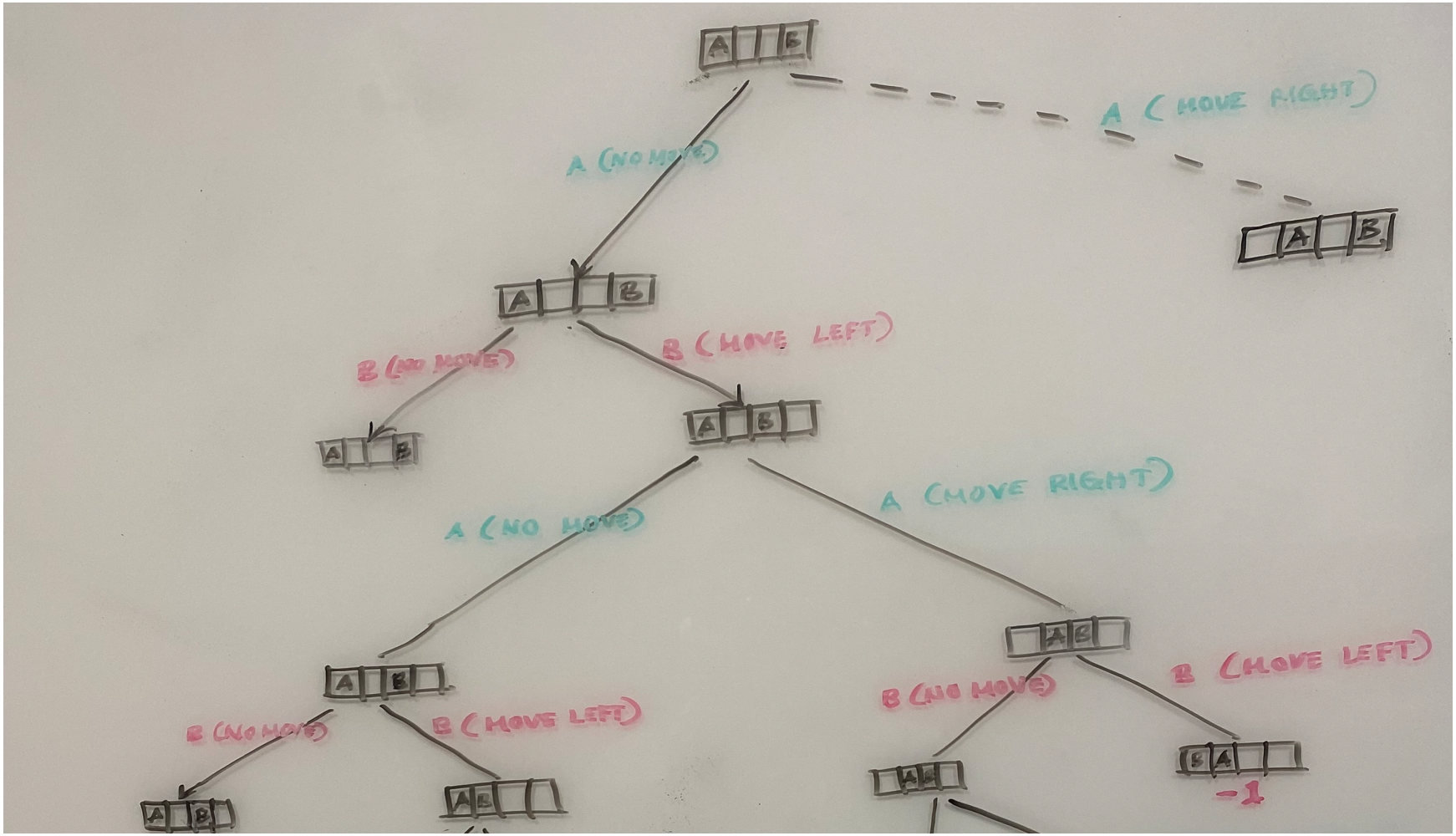
Tile Sliding Game Activity



Tile Sliding Game Activity



Tile Sliding Game Activity



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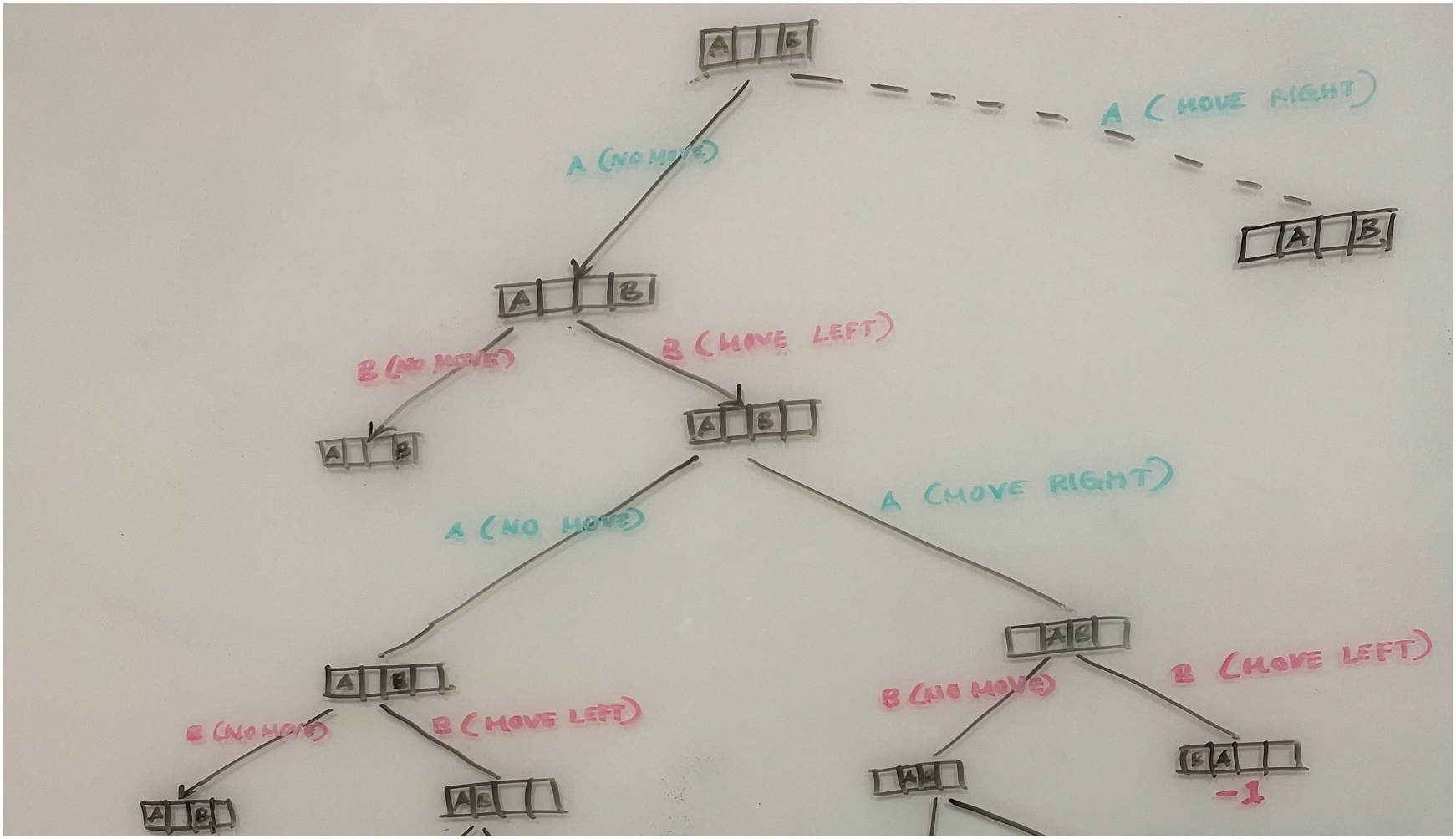
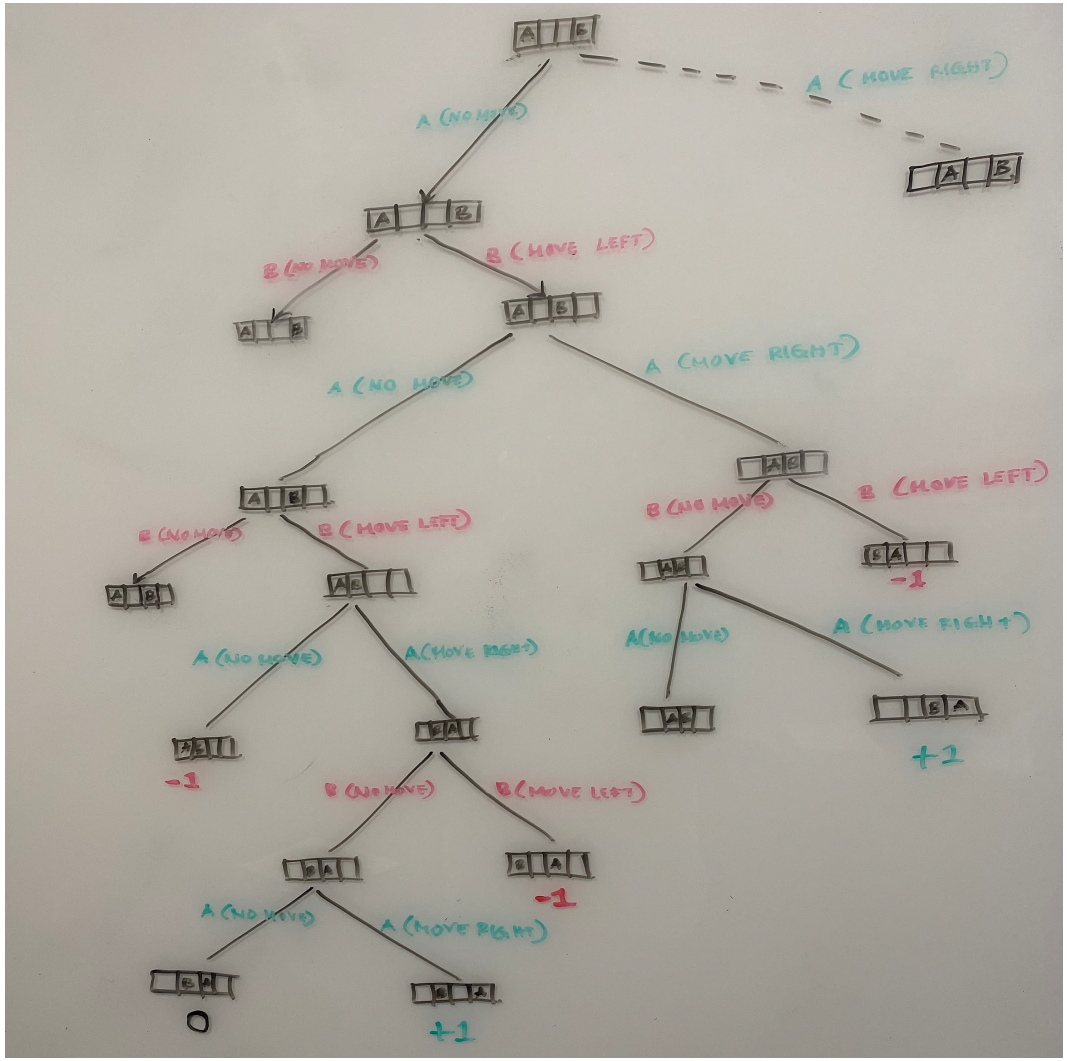


Tile Sliding Game Activity



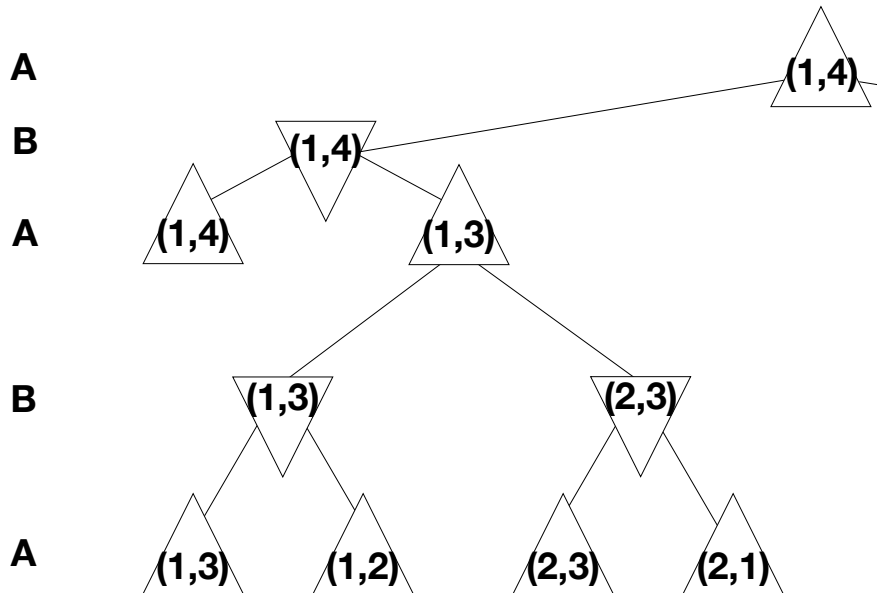
Tile Sliding Game Activity

- From the above configuration, play a game with a partner and draw the game tree
  - Can either player guarantee victory?
  - Can one of the players force a tie?
  - How do you tell this from the game tree?
  - What move should A make on her first move?

Tile Sliding Game Activity

- Simplified representation of the game states:
  - You can represent each state of this game as (n_A, n_B)
  - n_A, n_B are both numbers between 1 and 4
  - n_A is the position of the Player A
  - n_B is the position of the Player B

Class Activity



Properties of MINIMAX Algorithm

- Minimax performs a full D*epth-First Search (DFS)* of the game tree
- Suppose the maximum depth of the search tree is m and the number of choices at each step is at most b
- Time complexity? $O(b^m)$
- Space complexity? $O(bm)$
- For chess, $b \approx 35$, $m \approx 100$ for "reasonable" games. Exact solution completely infeasible

Issues

- The issue with minimax search is the number of game states it has to examine is exponential in the depth of the tree
- Can we feasibly look at every node in the tree?
- How do we find shortcuts to save computation?
- We will consider these questions going forward, but for the time being, we will consider examples of the minimax algorithm in action